Frequency Domain Design of a Complex Controller under Constraints on Robustness and Sensitivity to Measurement Noise

Tomislav B. Šekara, Miloš B. Trifunović and Vidan Govedarica

Abstract—New general rules have been developed for designing complex controllers under constraints on robustness and sensitivity to measurement noise. The design is based on a compromise between robustness and performance. This solution makes possible obtaining practically realizable complex controllers. It is shown that the proposed method results into considerably better performance and robustness indices, compared to those obtained by the optimal PID controller.

Index Terms—Complex controller, PID controller, Robustness, Frequency domain.

I. INTRODUCTION

The paper presents general rules for designing a complex controller $C(s)$, intended for the processes involving time delays and having an arbitrary order and multiple astaticism. The problem of control of complex processes (multiple instabilities, multiple astaticism, dominant time delay) cannot be solved adequately by applying PID controllers, which is the main reason for developing the methods for design of complex controllers.

It is well known that about 94% of feedbacks in industry are realized by PI/PID controllers [1], while in petrochemical industry this percentage is 97% [2,3].

Owing to a high significance of PI/PID, very efficient and simple procedures for tuning parameters of industrial controllers have been developed [4,5,6] as well as optimization procedures [7-22] for designing PI/PID controllers so the IAE (Integral of Absolute Error) is minimized under constraints on robustness, which satisfies the criterion defined in [23].

In addition to the mentioned methods, there are methods for design of PID controllers which are based on the IMC (Internal Model Control) controller [24-26]. The IMC method of controller design contains one adjustable parameter $\lambda$, for a narrow class of processes, has direct influence on the time constant of the closed loop system.

Response of a process regulated by applying an IMC method to a Heaviside-type disturbance is dependent on the dominant dynamics of the process. E.g., if a process is dominated by oscillatory dynamics, responses to any disturbance will be oscillatory.

For the purpose of accomplishing adequate indices of robustness and performance for a wider class of stable and unstable processes, new methods of designing complex controllers based on the modified IMC structure [27-29] have been developed. However, the design rules for complex controllers by applying these methods have not been designed for the general form of transfer function of the process $G_p(s)$, but only for certain classes of processes $G_p(s)$ [27-29].

Complex controller $C(s)$ defined in this work is designed for the general form of transfer function of the process, given in a rational form including delay $G_p(s)=H(s)\exp(\cdot ts)/Q(s)$ under constraints on robustness and sensitivity to measurement noise.

The adjustable parameters of the complex controller $C(s)$ are the time constant $\lambda$ and relative damping factor $\zeta$ of the dominant poles of the process in the closed loop with the complex controller $C(s)$ [7]. By adjusting parameter $\zeta$ one can accomplish a compromise between the robustness and performance indices, which is not possible for complex controllers designed by IMC [24-26] or modified IMC [27-289].

The proposed complex controller $C(s)$ is compared with the PID controller [7] through a series of simulations of a wide class of industrial processes. It is shown that the application of the proposed method results in considerably better indices of robustness and performance compared to those obtained with the method described in [7].

II. DESIGN OF THE COMPLEX CONTROLLER FOR A PROCESS HAVING TRANSFER FUNCTION $G_p(s)$

The control structure involving complex controller $C(s)$ is presented in Fig. 1.

![Fig. 1. The control structure.](image-url)
In general, \( G_p(s) = H(s) e^{-r_t} / Q(s) \), where \( Q(s) \) and \( H(s) \) are polynomials of the order \( \deg Q(s) = n \geq \deg H(s) = m \) and \( H(0)=h_0 \neq 0 \). In order to facilitate the process of deriving, with no loss in generality, it is assumed that \( \deg H(s) = 0 \), i.e., \( H(s)=h_0 \). The complementary sensitivity function of the controlled process \( G_p(s) \) of Fig.1 is given by relation 

\[
 T_p(s) = L(s) / (1+L(s)) ,
\]

with \( p \geq 2n, \ p,n \in N \) and adjustable parameters \( \lambda_i > 0, k=1,p, \eta_j \in R, j=1,n \), which are determined on the basis of the desired performance of the closed loop system. From relations (1), the controller \( C(s) \) of the process having transfer function \( G_p(s) \) resulting in maximum suppression of disturbance \( d \) or \( n \) is

\[
 C(s) = \frac{1}{h_0} \frac{T(s)}{1-T(s)} = \frac{1}{h_0} \frac{Q(s)N(s)}{F(s)} ,
\]

where \( F(s) = P(s) - e^{-r_t} N(s) \).

In general, parameters \( \eta_i, \eta_n \) are determined so that the poles of process \( G_p(s) \) are cancelled by the zeros of function \( F(s) \). Let the poles of process \( G_p(s) \) be: \( s_1=s_2=...=s_v=0 \), \( s_{v+1}=s_{v+2}=...=s_{v+r} \neq 0 \), i.e. zeros of polynomial \( Q(s) \), of the order \( v \) and \( r \). Let the remaining zeros of polynomial \( Q(s) \) be simple, then parameters \( \eta_i, \eta_n \) are determined according to the following rules.

**Rule 1.** If the zeros of polynomial \( Q(s) \) are \( s_1=s_2=...=s_v=0 \), parameters \( \eta_i, \eta_n \) are determined from condition

\[
 \frac{d^j F(s)}{ds^j} \bigg|_{s=0} = 0 , \ j = 1,v .
\]

**Rule 2.** If the zeros of polynomial \( Q(s) \) are \( s_{v+1}=s_{v+2}=...=s_{v+r} \neq 0 \), parameters \( \eta_{v+1}, \eta_r \) are determined from condition

\[
 \frac{d^{j-1} F(s)}{ds^{j-1}} \bigg|_{s=s_{v+r}} = 0 , \ j = 1,r .
\]

**Rule 3.** If the zeros of polynomial \( Q(s) \) are simple \( s_{v+1}, s_n \), parameters \( \eta_{v+1}, \eta_n \) are determined from condition

\[
 F(s) \bigg|_{s=s_{v+r}} = 0 \quad \text{for} \quad j = 1,n-v-r .
\]

Polynomial \( P(s) \) is usually taken in the form

\[
 P(s) = (\lambda s + 1)^p .
\]

For the purpose of achieving better compromise performance/robustness in this work a new form of polynomial \( P(s) \) is proposed

\[
 P_1(s) = (\lambda^2 s^2 + 2\xi \lambda s + 1)^p , \ \xi \in O(1) , \ p \geq 1 .
\]

**Rule 4.** If in Rule 2 or Rule 3 some of the zeros of polynomial \( Q(s) \) has a positive real part (unstable process), in controller (2) canceling of these zeros in the denominator and zeros in the numerator has to be carried out (elimination of dipoles).

**Remark 1.** If \( \deg H(s) > 0 \), the relations given by (1) - (7) remain the same and polynomial \( N(s) \) in (1) becomes

\[
 N(s) = (\eta_i s^n + \eta_{n-i} s^{n-1} + ... + \eta_1 s + 1) H(s) / h_0 , \ h_0 \neq 0 , \ N(0) = 1 .
\]

On the basis of (7), free parameters of the complex controller (2) are the time constant \( \lambda > 0 \) and relative damping factor \( \xi > 0 \) of the closed loop system, like in [7]. The damping factor which is introduced in the design of complex controller plays a significant role in accomplishing a compromise between the performance and robustness indices. It is shown later that through the damping factor one can exert influence upon sensitivity to measurement noise at high frequencies \( M_n \)

\[
 M_n = \lim_{\omega \to \infty} \frac{C(i\omega)}{1+C(i\omega)G_p(i\omega)} .
\]

In order to strike a compromise between desired performance IAE and \( M_n = \max_{\omega} \Rightarrow (1+L(i\omega)) \), time constant \( \lambda \) should satisfy condition

\[
 \max_{\omega} \frac{1}{1+C(i\omega)G_p(i\omega)} = M_n .
\]

For given \( \zeta \) and \( M_n \), time constant \( \lambda \) is determined by solving two nonlinear algebraic equations like in [7].

\[
 \frac{1+C(i\omega)G_p(i\omega)}{\partial i\omega} = 0 ,
\]

Initially, parameter \( \xi \) should be taken as \( \xi = 1 \) and parameter \( \lambda \) close to the estimated transport delay. By determining time constant \( \lambda \) for different values of parameter \( \xi \), one accomplishes a compromise between the values IAE, \( M_n \) and \( M_p \). A comparison of the qualities of control for different values of parameter \( \zeta \) is analyzed in detail in the next section.

**III. COMPARATIVE ANALYSIS AND SIMULATIONS**

A comparison of the proposed method for design of controller \( C(s) \) (2) for different values of parameter \( \xi \) is given in Table 1 for sixteen representative typical dynamic characteristics:

\[
 G_{p_1}(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}, \ G_{p_2}(s) = \frac{1}{(s+1)^r} .
\]

...
\[ G_{p3}(s) = \frac{1}{\prod_{k=0}^{3}(0.7^k s + 1)}, \quad G_{p4}(s) = \frac{e^{-5s}}{(s + 1)^2}, \quad G_{p5}(s) = \frac{1-s}{(s+1)^3}, \]

\[ G_{p6}(s) = \frac{1}{\prod_{k=0}^{3}(0.2^k s + 1)}, \quad G_{p7}(s) = \frac{(2s+1)e^{-4s}}{(10s+1)(7s+1)(3s+1)}, \]

\[ G_{p8}(s) = \frac{-(13.81s+1)(18.4s+1)}{(59s+1)^2}, \quad G_{p9}(s) = \frac{e^{-s}}{(s^2+0.1s+1)}, \]

\[ G_{p10}(s) = \frac{e^{-0.5s}}{s}, \quad G_{p11}(s) = 1, \quad G_{p12}(s) = \frac{1}{s(s+1)}, \]

\[ G_{p13}(s) = 4e^{-2s} \frac{e^{-0.5s}}{s}, \quad G_{p14}(s) = \frac{(5s-1)(2s+1)(0.5s+1)}{e^{-0.5s}(s+1)^2}, \]

\[ G_{p15}(s) = \frac{e^{-0.1s}}{(s-1)^2}, \quad G_{p16}(s) = \frac{e^{-0.5s}}{s}. \]

The controllers for all processes except the unstable ones \((G_{p13}(s), G_{p14}(s), G_{p15}(s))\) of the form (2) with the corresponding parameters \(\lambda\) and \(\zeta\) from Table 1. For unstable processes, on the basis of rule 4, in order to eliminate unstable dipoles of the controller, time delay \(e^{-\tau}\) in polynomial \(F(s)\) is approximated by Pade approximation of the order \(N/N\), where \(N\) is chosen so that the robustness and performance indices are preserved. For processes \(G_{p12}(s)\) and \(G_{p13}(s)\) it is sufficient to take \(N=2\)

\[ e^{-\tau} = \frac{12 - 6\tau s + \tau^2 s^2}{12 + 6\tau s + \tau^2 s^2}, \]  

i.e. \(N=3\) for process \(G_{p13}(s)\)

\[ e^{-\tau} = \frac{120 - 60\tau s + 12\tau^2 s^2 - \tau^3 s^3}{120 + 60\tau s + 12\tau^2 s^2 + \tau^3 s^3}. \]

E.g., on the basis of (1) – (7), general form of the controller (2) for unstable process \(G_{p13}(s)\) is given by

\[ C(s) = \frac{(4s-1)(\eta s+1)}{4((\lambda^2 s^2 + 2\zeta\lambda s + 1)-e^{-2s}(\eta s+1))}. \]

For \(\lambda=2.335\) and \(\zeta=1\) obtained on the basis of relations (9)-(11) and replacing transport delay \(e^{-\tau}\) by approximation (13) for \(\tau = 2\) one obtains the controller

\[ C(s) = \frac{2.3(s+2.322)(s+0.0797)(s+0.25)(s^2 + 3.678s + 6.459)}{s(s+9.186)(s+0.2203s + 7.0334)} \]

and upon canceling the dipole \(s = 0.25\), the final transfer function of controller from Table 1 for process \(G_{p13}(s)\) is

\[ C(s) = \frac{2.3(s+2.322)(s+0.0797)(s^2 + 3.678s + 6.459)}{s(s+9.186)(s^2 + 0.2203s + 7.0334)}. \]

By using the equivalent procedure, all other controllers of unstable processes have been determined, with approximation (12) applied for processes \(G_{p14}(s)\) and \(G_{p15}(s)\).

In order to reduce the order of the controllers of stable processes obtained by applying rules 1 to 3, it is desirable to apply the described reduction by cancellation of dipoles. For stable processes having dominant delays, this reduction of a complex controller may lead to degradation of the robustness and performance indices, thus this reduction is not recommendable. Such an example is process \(G_{p14}(s)\).

From Table 1 it is clear that for all processes \(G_{p3}(s), j=1,...,16\), when applying \(C(s)\) for the same \(M_s\), the adjustable parameter \(\zeta\) allows accomplishing a compromise between IAE, \(M_n\), and \(M_p\). This parameter is of key significance, since by its use one can decrease or increase value of \(M_s\) and improve the robustness and performance indices (Figs. 2 and 3).
The proposed method for design of the complex controller $C(s)$ (2) will be compared to the PID controller [7], which, as has been shown in [7], accomplished the robustness and performance indices the same as the optimal PID [21]. The comparison of these methods, assuming the same values of $M_f$ and $M_m$, is presented in Table 2 for the processes $G_p(s), j=1,...,16$.

In Table 2, parameter $\zeta$ for all complex controllers has been determined so that practically the same value of $M_d$ as in the case of the PID controller is obtained. It can also be seen from Table 1 that the complex controller ensures a considerably better quality of control compared to that of the PID controller.

The following figures show the response to a Heaviside type of disturbance of the complex controller $C(s)$ and PID controller [7].

From Figs. 4-7 it is obvious that application of the complex controller results in a significantly lower IAE, with practically the same robustness as the one obtained with the PID controller. For unstable processes the complex controller gives considerably higher indices of robustness and performance compared to those of the PID controller. It should be mentioned that for the processes of higher order of instability and astaticism, complex controllers can be successfully designed as demonstrated with processes $G_{p15}(s)$ and $G_{p16}(s)$.
Fig. 6 Response to a Heaviside-type disturbance of process $G_{p12}(s)$ in closed loop with controllers from Table 2 for $M_2=5.91$.

Fig. 7 Response to a Heaviside-type disturbance of process $G_{p13}(s)$ in closed loop with controllers from Table 2 for $M_3=2.41$.

IV. THE CONCLUSION

Design of complex controllers is aimed at increasing the robustness and performance indices compared to those obtainable with conventional controllers. For designing complex controllers an adequate knowledge of transfer function of the process is required. The paper presents general rules for designing complex controllers which have been tested on a wide class of processes. By applying suitable approximations of complex controllers, adequate conventional controllers are obtained for certain class of processes. The comparative analysis and simulations gave the expected results.

REFERENCES


