PI-like Observer Structures in Digitally Controlled DC Servo Drives: Theory and Experiments

Milica B. Naumović

Abstract—**This paper deals with the problem of the shaft speed estimation in a digitally controlled DC servo drive. Some different observer structures are presented and compared. The developed extended observers enable proper estimation of the plant state variables, even under the action of a constant or slow varying load torque disturbance. Two useful procedures for the adjustment of observer gains are proposed and verified by both numerical simulations and real-time experimental results.**

Index Terms—**Shaft speed estimation, Extended observer, Digitally controlled DC servo drive.**

I. Introduction

High-performance servo drives are required in many applications of digitally controlled machines. Two types of position sensors are most frequently applied: optical encoders (absolute or incremental), and electromagnetic resolvers (inherently absolute) [1], [2]. The transducer output signal is used as the position feedback signal in a position-controlled system; consequently, the signal in the inner velocity loop must be estimated. In speed-controlled high-performance servo drives, the feedback velocity signal is to be estimated from the torque command and measured angular position of the motor shaft, in the presence of the quantization noise and a constant or slow varying load torque disturbance. In order to obtain smooth and sufficiently accurate position and speed signals, the observer structure is often implemented.

This paper deals with the analysis and design of a dynamic system that is able to estimate state variables (position and speed signals) in an environment where the shaft position information is incomplete due to a limited resolution of the position transducer, even in the presence of a constant load torque disturbance. After comparing several different methods of velocity estimation, in this paper a novel approach to the extending of the discrete-time observer is proposed.

This paper is organized as follows. The problem formulation is given in Section II. Section III presents the design procedures of the observers extended by using the additionally introduced integral terms in a digitally controlled servo drive. Procedures for calculating the observer gains are given in Section IV. Section V gives a concrete example to demonstrate the effectiveness of the proposed observers. Finally, Section VI presents the concluding remarks.

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II. Problem Formulation

In all cases, no matter what type of sensor is utilized in the digitally controlled servo system, the shaft position is read as a digital signal. Hence, the resolution of the shaft position measurement is limited. Due to the finite resolution, the actual shaft position differs from the digital word representing the position (lower resolution - the larger difference).

To estimate velocity signal, the least complicated algorithm yields

$$\hat{\omega}(k) = \frac{\theta(k) - \theta(k-n)}{nT}, \tag{1}$$

where T is sampling period, θ is the angular position of drive shaft, k is the sample number index, and integer $n \geq 1$. By setting $n = 1$ the average velocity over the preceding sampling interval is estimated as the well-known Euler's approximation of the derivative, that is a simple first difference. Note that the velocity resolution is limited directly by the transducer resolution and the time interval nT.

Due to the finite resolution of the angle measurement, the shaft speed signals estimated by (1) would be highly contaminated by the quantization noise. In order to improve the quality of the shaft velocity estimation, an observer structure is often implemented. Besides enabling an accurate state estimation of the control object, the applied observer can be used also for filtering the measurement noise.

Consider the discrete-time model of the plant

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{E}(T)\mathbf{x}(k) + \mathbf{F}(T)\mathbf{u}(k)\\ \mathbf{c}(k) &= \mathbf{D}\mathbf{x}(k) + \mathbf{H}\mathbf{u}(k)\end{aligned} \tag{2}$$

where $\mathbf{x}(k) = \mathbf{x}(kT) \in \mathbb{R}^n$ is the state vector to be observed, $\mathbf{u}(k) = \mathbf{u}(kT) \in \mathbb{R}^r$ and $\mathbf{c}(k) = \mathbf{c}(kT) \in \mathbb{R}^m$ are known control input vector and output vector, respectively. The sampling interval is T. Constant matrices $\mathbf{E}$, $\mathbf{F}$, $\mathbf{D}$ and $\mathbf{H}$ have appropriate dimensions; the pair $(\mathbf{E}, \mathbf{F})$ is controllable and the pair $(\mathbf{E}, \mathbf{D})$ is observable.

The observer or asymptotic state estimator is a dynamic system, with inputs $\mathbf{u}(k)$, $\mathbf{c}(k)$ and output $\hat{\mathbf{x}}(k)$, having the property $\lim_{k\to\infty} \tilde{\mathbf{x}}(k) = \mathbf{0}$, where

$$\tilde{\mathbf{x}}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k) \tag{3}$$

is the estimation error.

The system plant and the associate observer are shown in Fig. 1. We adopt the notation

$$\mathbf{G}(z) := \left[\begin{array}{c|c} \mathbf{E} & \mathbf{F} \\ \hline \mathbf{D} & \mathbf{H} \end{array}\right] . \tag{4}$$

where

$$G(z) = \mathbf{D}\left(z\mathbf{I} - \mathbf{E}\right)^{-1}\mathbf{F} + \mathbf{H}, \quad z - \text{complex variable,} \tag{5}$$

is the transfer function matrix derived from (2).

The state variables reconstructed by the observer can be used by the control law

$$\mathbf{u}(k) = -\mathbf{K}\,\hat{\mathbf{x}}(k) . \tag{6}$$

Notice that the observer has a special structure

$$\begin{aligned} &\underbrace{\hat{\mathbf{x}}(k+1) = \mathbf{E}\hat{\mathbf{x}}(k) + \mathbf{F}\mathbf{u}(k)}_{apriori\ estimate} + \mathbf{G}\underbrace{\{\mathbf{c}(k) - \overbrace{[\mathbf{D}\hat{\mathbf{x}}(k) + \mathbf{H}\mathbf{u}(k)]}^{estimate\ of\ output}\}}_{output\ estimation\ error} \\ &\hat{\mathbf{x}}(k) = \mathbf{x}_0(k) . \end{aligned} \tag{7}$$

In order to obtain a state-space description for the system in Fig. 1 supplemented with the state feedback (6), we define the state vector to be $\left[\mathbf{x}(k) \mid \tilde{\mathbf{x}}(k)\right]^{\mathrm{T}}$, giving the equation for the closed-loop regulator system as follows

$$\begin{bmatrix} \mathbf{x}(k+1) \\ \tilde{\mathbf{x}}(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{E}-\mathbf{FK} & \mathbf{FK} \\ \mathbf{0} & \mathbf{E}-\mathbf{GD} \end{bmatrix}}_{\mathbf{E_c}} \begin{bmatrix} \mathbf{x}(k) \\ \tilde{\mathbf{x}}(k) \end{bmatrix} . \tag{8}$$

Because the matrix $\mathbf{E_c}$ is block upper-triangular, its eigenvalues are just the eigenvalues of the diagonal blocks. Thus we have that

$$\operatorname{eig}\left(\mathbf{E_c}\right) = \operatorname{eig}\left(\mathbf{E}-\mathbf{FK}\right) \cup \operatorname{eig}\left(\mathbf{E}-\mathbf{GD}\right), \tag{9}$$

where the symbol $\cup$ denotes the union. Equation (9) is known as the separation principle, which refers to the fact that the state feedback vector and the observer gains vector can be calculated separately [3]-[5].

In the case $\mathbf{H} = \mathbf{0}$ we can rewrite (7) as follows

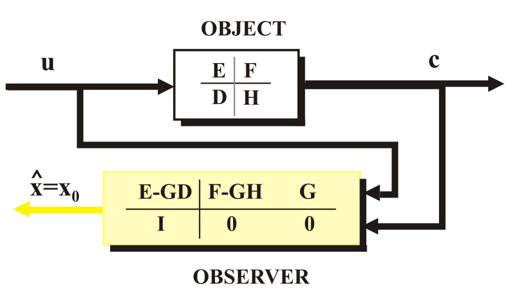


Fig. 1. Control Object and Observer.

$$\hat{\mathbf{x}}(k+1) = \left(\mathbf{E} - \mathbf{GD}\right)\hat{\mathbf{x}}(k) + \mathbf{F}\mathbf{u}(k) + \mathbf{G}\mathbf{c}(k) , \tag{10}$$

where all state values are estimated with the resolution limited only by the word length of the digital controller. The observer gain matrix $\mathbf{G}$ in (7) is to be determined according to requirements for the desired speed of estimation.

Recall, for the sake of simplicity, instead of identity observer, the reduced-order one is proposed to estimate only the unmeasured states. In the case of reduced-order observer design, the object model, like (2), becomes

$$\begin{aligned} &\begin{bmatrix} \mathbf{x}_a(k+1) \\ \mathbf{x}_b(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{aa} & \mathbf{E}_{ab} \\ \mathbf{E}_{ba} & \mathbf{E}_{bb} \end{bmatrix} \begin{bmatrix} \mathbf{x}_a(k) \\ \mathbf{x}_b(k) \end{bmatrix} + \begin{bmatrix} \mathbf{F}_a \\ \mathbf{F}_b \end{bmatrix} \mathbf{u}(k) \\ &\mathbf{c}(k) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_a(k) \\ \mathbf{x}_b(k) \end{bmatrix} , \end{aligned} \tag{11}$$

where $\mathbf{x}_a(k)$ is the state vector portion directly measured, and $\mathbf{x}_b(k)$ is the remaining portion to be estimated. Then the observer model takes the form as follows [3], [4]

$$\begin{aligned} \hat{\mathbf{x}}_b(k+1) = &\left(\mathbf{E}_{bb} - \mathbf{G}\mathbf{E}_{ab}\right)\hat{\mathbf{x}}_b(k) + \left(\mathbf{E}_{ba} - \mathbf{G}\mathbf{E}_{aa}\right)\mathbf{c}(k) \\ &+ \mathbf{G}\mathbf{c}(k+1) + \left(\mathbf{F}_b - \mathbf{G}\mathbf{F}_a\right)\mathbf{u}(k) . \end{aligned} \tag{12}$$

As it is well known, in the presence of constant or slow varying disturbances that may not be considered as state disturbances, both the identity observer (10) and the reduced-order observer (12) are not able to estimate the proper values of the state variables. Some modifications, that are based on the special extending the standard observer structures with the integral action [6]-[11], provide the correct estimation even under the disturbance action.

III. Position and Velocity Observation in a Digitally Controlled Servo Drive

By assuming the state vector as $\mathbf{x}(k) = \left[\theta(k) \quad \omega(k)\right]^{\mathrm{T}}$, where $\theta(k)$ and $\omega(k)$ are the shaft angular position and speed respectively, the state-space model of the object in Fig. 2 becomes

$$\begin{aligned} &\mathbf{x}(k+1) = \mathbf{E}(T)\mathbf{x}(k) + \mathbf{f}(T)u(k) \\ &c(k) = \mathbf{d}\mathbf{x}(k) \end{aligned} \tag{13}$$

with

$$\begin{aligned} &\mathbf{E}(T) = \begin{bmatrix} 1 & T_m\left(1 - \mathrm{e}^{-T/T_m}\right) \\ 0 & \mathrm{e}^{-T/T_m} \end{bmatrix} = \begin{bmatrix} 1 & e_1 \\ 0 & e_2 \end{bmatrix}, \\ &\mathbf{f}(T) = \begin{bmatrix} K_m\left(T + T_m\,\mathrm{e}^{-T/T_m} - T_m\right) \\ K_m\left(1 - \mathrm{e}^{-T/T_m}\right) \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 1 & 0 \end{bmatrix} . \end{aligned} \tag{14}$$

In above equations K_m and T_m are the gain factor and the mechanical time constant of the considered drive, respectively.

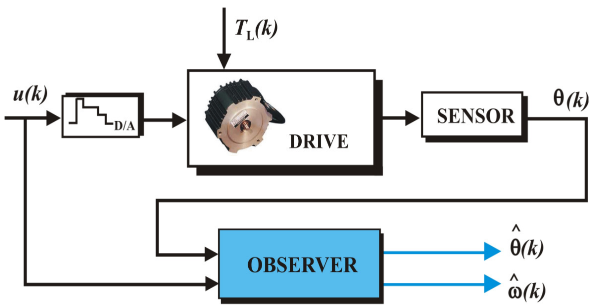


Fig. 2. Block diagram of a drive with observer.

The observer that joins the drive in Fig. 2 estimates the states $x_1(k)$ and $x_2(k)$ by using the control variable $u(k)$ and measured output $\theta(k)$ as inputs. Equations (10) and (12) of the ordinary identity observer and the reduced-order observer can be rewritten as

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} 1-g_1 & e_1 \\ -g_2 & e_2 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} u(k) + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} c(k) \quad (15)$$

and

$$\hat{x}_2(k+1) = \left(e_2 - g_2 e_1\right)\hat{x}_2(k) + g_2 c(k+1) - g_2 c(k) + \left(f_2 - g_2 f_1\right)u(k). \quad (16)$$

In numerous applications, as in robotics for example, the employed observer must enable the estimation of the plant state variables even under the action of the constant (gravitation) or a slow varying load torque disturbance $T_L(k)$. As it is well known [3], in the presence of such disturbances, that may not be considered as initial state disturbances, the ordinary observers (full or reduced-order) are not able to estimate the state variables.

For proper state estimation, the possibility of extending the ordinary discrete-time observer with additional integrators is discussed. The solution consists in the following: the observer gains multiply the generated errors of the state variables $\tilde{x}_i = x_i(k) - \hat{x}_i(k)$, $i = 1, 2$; the errors of position and velocity estimation are simultaneously processed through two discrete integrators assuring the zero steady-state estimation errors in the presence of a constant load torque disturbance T_L.

It is possible to choose the estimates of new state variables $\hat{x}_3(k)$ and $\hat{x}_4(k)$ to be the outputs of the inserted integrators resulting in the following discrete-time new state equations [7]

$$\hat{x}_3(k+1) = \hat{x}_3(k) + g_3\left[c(k) - \hat{x}_1(k)\right] = -g_3\hat{x}_1(k) + \hat{x}_3(k) + g_3 c(k) \quad (17)$$

and

$$\begin{aligned} \hat{x}_4(k+1) &= \hat{x}_4(k) + g_4\left[\frac{c(k)-c(k-1)}{T} - \hat{x}_2(k)\right] \\ &= -g_4\hat{x}_2(k) + \hat{x}_4(k) + \frac{g_4}{T}c(k) - \frac{g_4}{T}c(k-1)\ . \end{aligned} \quad (18)$$

After adding new state variables $\hat{x}_3(k)$ and $\hat{x}_4(k)$ into state vector to be observed $\hat{\mathbf{x}}(k) = \left[\hat{\theta}(k)\ \ \hat{\omega}(k)\right]^{\mathrm{T}}$, the vector difference equation of the identity observer (15) extended in the previously described manner becomes

$$\hat{\mathbf{x}}_{\mathrm{e}}(k+1) = \left(\mathbf{E}_{\mathrm{e}} - \mathbf{G}_{\mathrm{e1}}\mathbf{D}_{\mathrm{e}}\right)\hat{\mathbf{x}}_{\mathrm{e}}(k) + \mathbf{f}_{\mathrm{e}}u(k) + \mathbf{G}_{\mathrm{e2}}\begin{bmatrix} c(k) \\ c(k-1) \end{bmatrix}, \quad (19)$$

where

$$\hat{\mathbf{x}}_{\mathrm{e}}(k+1) = \begin{bmatrix} \hat{\mathbf{x}}(k+1) \\ \hdashline \hat{x}_3(k+1) \\ \hat{x}_4(k+1) \end{bmatrix}, \quad \mathbf{E}_{\mathrm{e}} = \left[\begin{array}{cc:cc} 1 & e_1 & 1 & 0 \\ 0 & e_2 & 0 & 1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right],$$

$$\mathbf{f}_{\mathrm{e}} = \begin{bmatrix} f_1 \\ f_2 \\ \hdashline 0 \\ 0 \end{bmatrix}, \quad \mathbf{D}_{\mathrm{e}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad (20)$$

$$\mathbf{G}_{\mathrm{e1}} = \begin{bmatrix} g_1 & 0 \\ g_2 & 0 \\ g_3 & 0 \\ 0 & g_4 \end{bmatrix}, \quad \text{and} \quad \mathbf{G}_{\mathrm{e2}} = \mathbf{G}_{\mathrm{e1}}\begin{bmatrix} 0 & 0 \\ 1/T & -1/T \end{bmatrix}.$$

Certainly, the described proportional double-integral observer (PI^{2}O) to be devised by using two additionally introduced integral terms of the output estimation errors can offer some degrees of freedom in the observer design.

In the case of the reduced-order observer we must return to (16) and introduce a new state variable $\hat{x}_4(k)$ as the output of the discrete integrator by which the observer is extended. In such a way obtained the reduced-order proportional integral observer (PIO) is described by the following equations:

$$\begin{aligned} \hat{x}_2(k+1) &= \left(e_2 - g_2 e_1\right)\hat{x}_2(k) + \hat{x}_4(k) \\ &\quad + g_2\left[c(k+1) - c(k) - f_1 u(k)\right] + f_2 u(k) \end{aligned} \quad (21)$$

$$\hat{x}_4(k+1) = -g_4\hat{x}_2(k) + \hat{x}_4(k) + \frac{g_4}{T}\left[c(k) - c(k-1)\right]\ .$$

IV. Procedures for Calculating the Observer Gains

A. *A Procedure for Parameter Adjustment of Reduced - Order PI Observer*

Recall, the unmeasured velocity variable can be proper estimated by the reduced-order PI observer even in the presence of constant or slow varying load torque disturbance

T_L acting on the drive of the system given in Fig. 2. The observer equations (21) can be rewritten in the form as

$$\begin{bmatrix} \hat{x}_2(k+1) \\ \hat{x}_4(k+1) \end{bmatrix} = \begin{bmatrix} e_2 - g_2 e_1 & 1 \\ -g_4 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_2(k) \\ \hat{x}_4(k) \end{bmatrix} + \begin{bmatrix} g_2 & 0 \\ 0 & g_4 \end{bmatrix} \begin{bmatrix} c(k+1) - c(k) - f_1 u(k) \\ c(k)/T - c(k-1)/T \end{bmatrix} + \begin{bmatrix} f_2 \\ 0 \end{bmatrix} u(k) . \tag{22}$$

The observer gains g_2 and g_4 are calculated according to the desired quality and speed of state estimation. The parameter setting may be conveniently performed assuming both observer poles to be the same and equal to

$$\sigma_z = \exp(-2\pi f_0 T) , \tag{23}$$

where f_0 denotes the observer bandwidth. Thus the characteristic equation of the extended observer given by (22) is

$$\det \begin{bmatrix} z - e_2 + g_2 e_1 & -1 \\ g_4 & z-1 \end{bmatrix} = 0 . \tag{24}$$

The desired characteristic equation, founded by multiplying out the observer poles, becomes

$$\left(z - \sigma_z\right)^2 = z^2 - 2\sigma_z z + \sigma_z^2 = 0 . \tag{25}$$

Hence, the simple relations, obtained by matching coefficients of like powers of z in (24) and (25), yield two unknown gains of PI observer as follows

$$\begin{aligned} g_2 &= (1 + e_2 - 2\sigma_z)/e_1 \\ g_4 &= (1 - \sigma_z)^2 \quad . \end{aligned} \tag{26}$$

B. A Procedure for Adjustment of PI² Observer Parameters

Let us denote by

$$\Delta_o = \det\left[z\mathbf{I} - \left(\mathbf{E}_e - \mathbf{G}_{e1}\mathbf{D}_e\right)\right] = z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0 = 0 \tag{27}$$

the characteristic equation of the extended identity observer, whose model is given by (19)-(20), and may be written in the form

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \\ \hat{x}_3(k+1) \\ \hat{x}_4(k+1) \end{bmatrix} = \begin{bmatrix} 1-g_1 & e_1 & 1 & 0 \\ -g_2 & e_2 & 0 & 1 \\ -g_3 & 0 & 1 & 0 \\ 0 & -g_4 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \\ \hat{x}_3(k) \\ \hat{x}_4(k) \end{bmatrix} + \begin{bmatrix} f_1 & g_1 & 0 \\ f_2 & g_2 & 0 \\ 0 & g_3 & 0 \\ 0 & g_4/T & -g_4/T \end{bmatrix} \begin{bmatrix} u(k) \\ c(k) \\ c(k-1) \end{bmatrix} . \tag{28}$$

The observer gain matrix $\mathbf{G}_{e1}$ in (20) is calculated according to the desired observer pole spectrum determining the speed of convergence between the state of the system and the state estimated by the observer. The setting of gains g_i , $i = 1,2,3,4$ may be conveniently performed assuming all observer poles to be the same and equal to $\sigma_z = \exp(-2\pi f_0 T)$, where f_0 denotes the observer bandwidth, as in (23). Therefore, the observer gains are calculated from

$$\det \begin{bmatrix} z-1+g_1 & -e_1 & -1 & 0 \\ g_2 & z-e_2 & 0 & -1 \\ g_3 & 0 & z-1 & 0 \\ 0 & g_4 & 0 & z-1 \end{bmatrix} = \left(z - \sigma_z\right)^4 . \tag{29}$$

Finally, after equating coefficients of like powers of z , the following relationships may be written

$$\begin{aligned} g_1 &= 3 - 4\sigma_z + e_2 \\ e_1 g_2 + g_3 + g_4 &= 3 + 6\sigma_z^2 + \left(e_2 - 4\sigma_z\right)\left(e_2 + 2\right) \\ \left(1 - e_2\right) g_3 + \left(3 + e_2 - 4\sigma_z\right) g_4 &= 4\left(1 - \sigma_z\right)^3 \\ g_3 \, g_4 &= \left(1 - \sigma_z\right)^4 \quad . \end{aligned} \tag{30}$$

V. ILLUSTRATIVE EXAMPLE

In this section an example of shaft velocity estimation in a low power DC motor servomechanism is presented. The desired quality of the transient response of the considered system is matched by the conventional controller whose parameters are calculated by using the standard pole placement method. In view of the fact that it is well-known from classical control theory, the design procedure of the controller is not explained in detail. The experimental setup described in this section has been built to demonstrate proper estimation of the plant state variables, even in the case of the actions of constant or slow varying load torque disturbances. The simulation results of the proposed observers studied in the previous section are compared with the experimental ones. The goal is to illustrate that in all cases the estimated and the experimentally obtained results are good matched.

A. The Experimental Setup

Fig. 3 visualizes the structure of the experimental environment for rapid control prototyping that was realized during the PhD thesis research in Control Engineering Laboratory at the University of Niš, Faculty of Electronic Engineering [12]. The experimental setup of the servo system consists of several functional elements as follows: **1.** DC motor with incremental encoder, **2.** Personal computer upgraded to a powerful development system for rapid control prototyping, **3.** Led panel for signal connection, **4.** PWM power amplifier, and **5.** Power supply.

As control object, a DC motor Type Bautz E586MGB is used with the following rating [12]: $M_{\max} = 0.22\,\text{Nm}$, $I_{\max} = 3.7\,\text{A}$, $n_{\max} = 6000\,\text{min}^{-1}$, the voltage constant $k_e = 5.85\ \text{V}/1000\,\text{min}^{-1}$, the torque constant $k_t = 0.056\,\text{Nm/A}$.

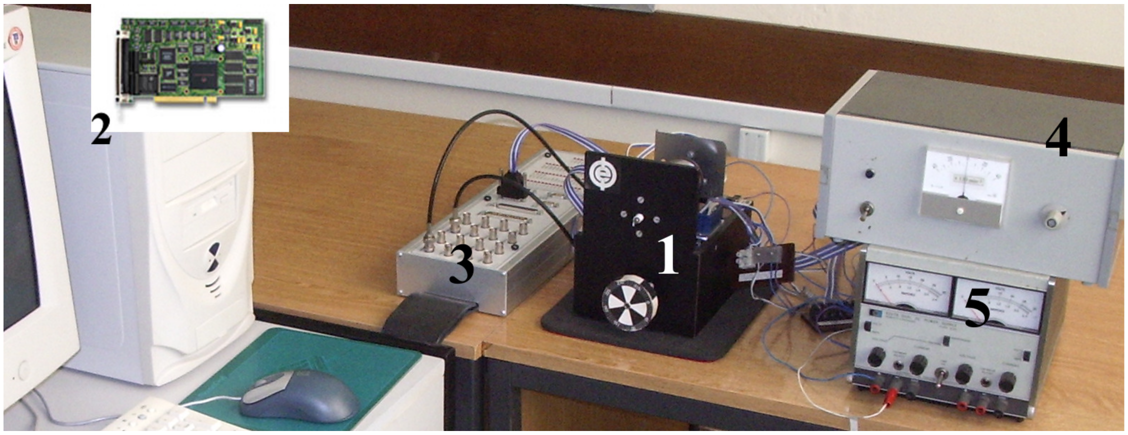


Fig. 3. Experimental setup [12].

For the position measurement $\theta(t)$ the quadrature incremental encoder with 1000 lines is used. The observer-based control algorithms are implemented by using the dSPACE system [13]. Namely, as a standard PC card, dSPACE DS1104 R&D Controller Board is slotted into PC using $5\,\mathrm{V}$ PCI bus as a backplane, upgrading in that way the personal computer to a powerful development system for rapid control prototyping.

The controller board is based on the Motorola 32-bit floating-point MPC 8240 processor with $250\,\mathrm{MHz}$ CPU, and contains all the necessary peripherals for AC and DC motor control in various fields. Moreover, because of demanding I/O operation, a slave DSP subsystem based on the Texas Instruments TMS320F240 16-bit fixed point digital signal processor with $20\,\mathrm{MHz}$ clock frequency is provided. Led panel CLP1104 indicates the status of the board's digital signals. Interfacing the computer DS1104 controller board with the control object input and output, as well as indication the status of the board's digital signals is possible through the LED panel CLP1104.

Using the information related to the measured angular position of the motor shaft $\theta(t)$ and the reference signal $\theta_{\mathrm{r}}(t)$, the dSPACE system generates, based on the implemented control algorithm, a control signal which after PWM power amplifier with the carrier frequency of 15 kHz produces a voltage to the motor.

dSPACE Prototyper is a flexible development system that enables rapid control design of the real controlled system without manual programming. Namely, the dSPACE Real-Time Interface allows to implement the considered MATLAB®/Simulink model onto dSPACE hardware via code generated by Real-Time Workshop automatically. This software offers an application that makes observation of the processed variables in real time possible.

B. Simulation and Experimental Results

In this section the simulation and experimental results for the observers studied in the previous section are presented. To verify the usefulness of suggested procedures for setting of observer gains, ensuring the proper speed estimation of the drive given in Fig. 3, the system simulation has been carried out in all details, taking into account the limited resolution (the increment of $2\pi/4000\text{ rad}$) of position sensor.

The electrical subsystem dynamics of the motor and the inertial dynamics of the power amplifier can be neglected. The plant in this example is a type-1 servo with transfer function from input current to output angular position as follows

$$G_{\mathrm{p}}(s)=\frac{K_m}{s(T_m s+1)}\ . \tag{31}$$

The motor's gain factor $K_m = 24.8$ and the mechanical time constant $T_m = 0.0379\text{ s}$ are computed on the basis of the experimentally recorded open-loop step response given in Fig. 4. Note, that due to the finite resolution of angle measurement, the shaft speed signals estimated by (1), are contaminated by the quantization noise, especially in the case $n = 1$.

The sampling period $T = 0.001\text{ s}$ was adopted. The speed of continuous-time closed-loop system response and stability margin are specified by the dominant pole pair (the damping ratio $\zeta = 0.707$, and the natural frequency $\omega_n = 10\text{ rad/s}$) located in Nyquist frequency region. The desired quality of transient response is matched by the gains of the position PI regulator $K_p = 0.52024$ and $K_I = 0.0012885$.

According to relations (15) and (16), the gains of the ordinary identity and the reduced-order digital observers were adjusted to values given in Table I, insuring the bandwidth of 4.5 Hz and proper speed estimation. Also, the gains for both digital reduced-order PI observer and full-order PI^2 observer

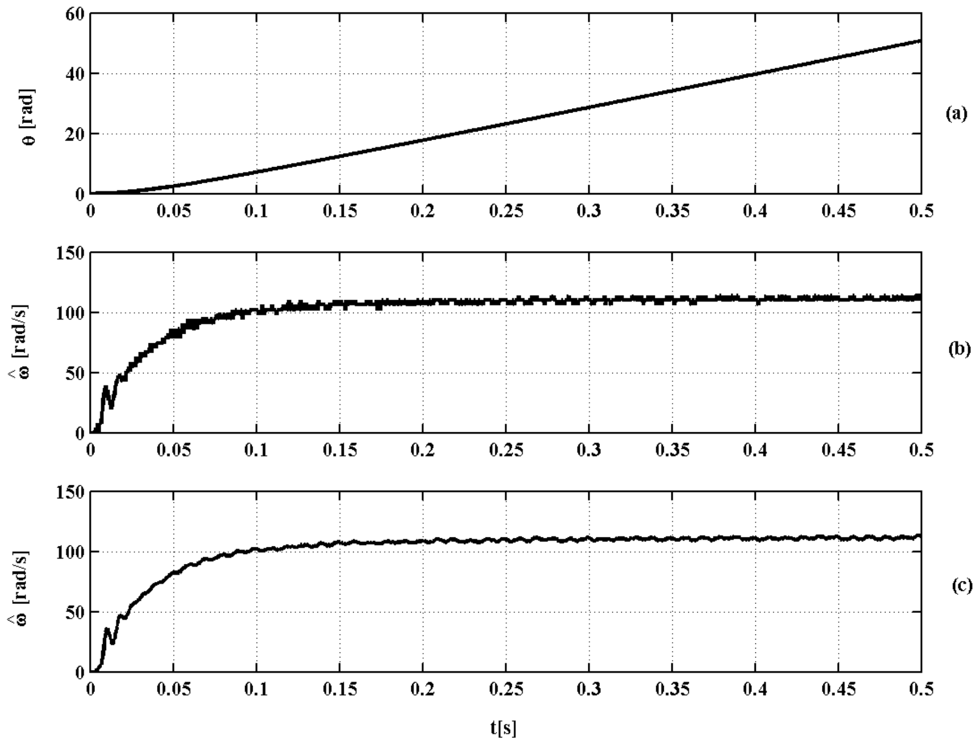


Fig. 4. (a) Open-loop step response $\theta(t)$; (b) Estimate of shaft speed $\hat{\omega}(t)$ derived by Euler's approximation of the derivative; (c) Estimate of shaft speed $\hat{\omega}(t)$ derived by simple algorithm (1) and $n = 5$.

were set according to relations (26) and (30) to values given in Table I, providing the same transient behavior.

In the simulations, as well as in the experiment the system was excited by the step reference signal $\theta_{\mathrm{ref}}(t) = 10\,\mathrm{h}(t-2)$ rad , and by the external disturbance over the period 6 to 10 seconds. The disturbance was a constant load torque $T_{\mathrm{L}} = 0.1$ Nm , which is 53% of the rated torque. During the control object modeling the electrical time constant was neglected, and the effect of disturbance can be mapped onto the object input, and presented by the appropriate voltage signal $M_{\mathrm{o}}^{*} = 3.82$ V which acts inside the control channel.

Figs. 5-7 and Figs. 8-10 compare experimental versus full-model simulation results for both the reduced-order observer and the identity observer, respectively. These results show a remarkable agreement between the simulated and measured quantities of the considered system.

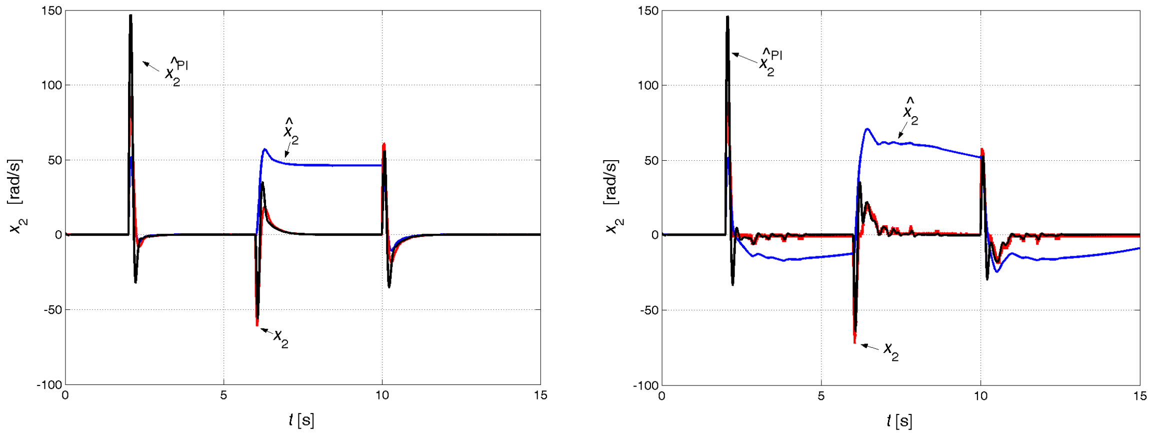
Note that the control object is low power DC motor with some dry friction problems which are especially expressive in the case of different positioning tasks.

The results presented in Figs. 5-7 indicate that under relatively unfavorable real conditions the proper shaft speed estimation can be provided by using the reduced-order PI observer whose setting is proposed in the previous section. The same conclusion can be drawn about the full-order PI^2 observer from the results given in Figs. 8-10.

Some differences between simulation and experimental results, and the presence of the observation errors, shown in Fig. 11, are caused by the final resolution of the applied encoder, the quantization noise of the digital hardware, as well as by the unmodeled dynamics.

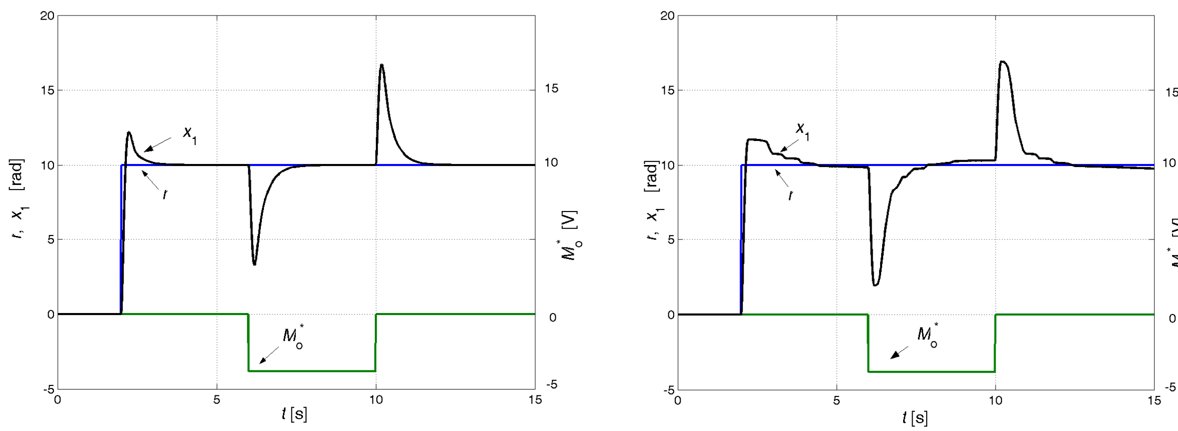
TABLE I
OBSERVERS SETTING

Type of Observer Structure	Observer Gains			
	g_1	g_2	g_3	g_4
Identity Observer	0.0301626	0.00659052	-	-
PI^2 Observer	0.0853859	0.92137800	0.000762402	0.000762402
Reduced-Order Observer	-	2.58350000	-	-
PI Observer	-	30.5470000	-	0.000762402



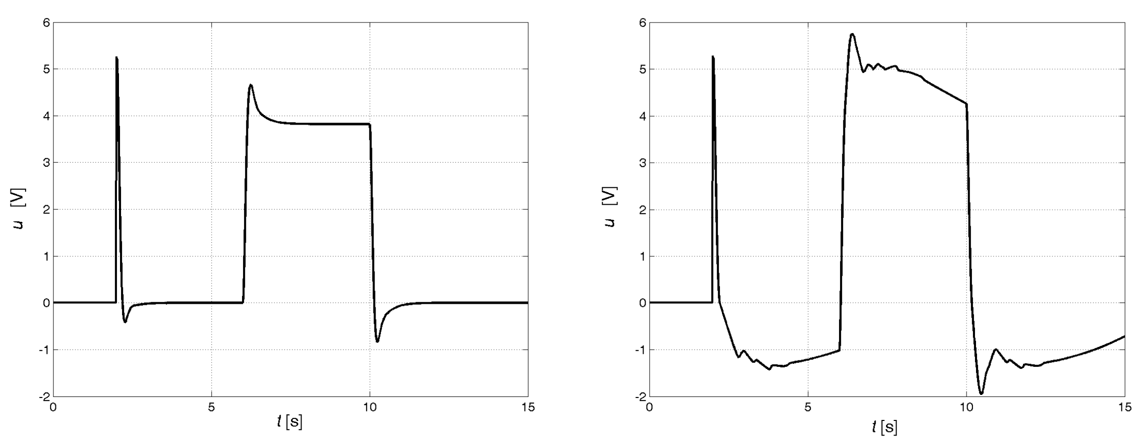
(a) (b)

Fig. 5. True and estimation values of the shaft speed using reduced-order observer (16) and reduced-order PI observer (22) (a) simulation, (b) experiment.



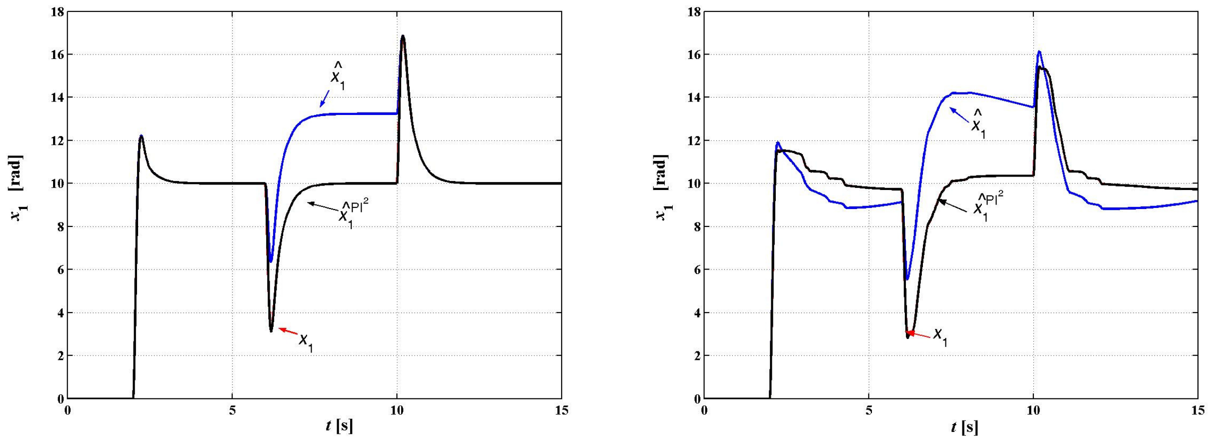
(a) (b)

Fig. 6. Reference position (r), step response of shaft position (θ) and on the plant input mapped load torque $\left(M_O^*\right)$ (a) simulation, (b) experiment.



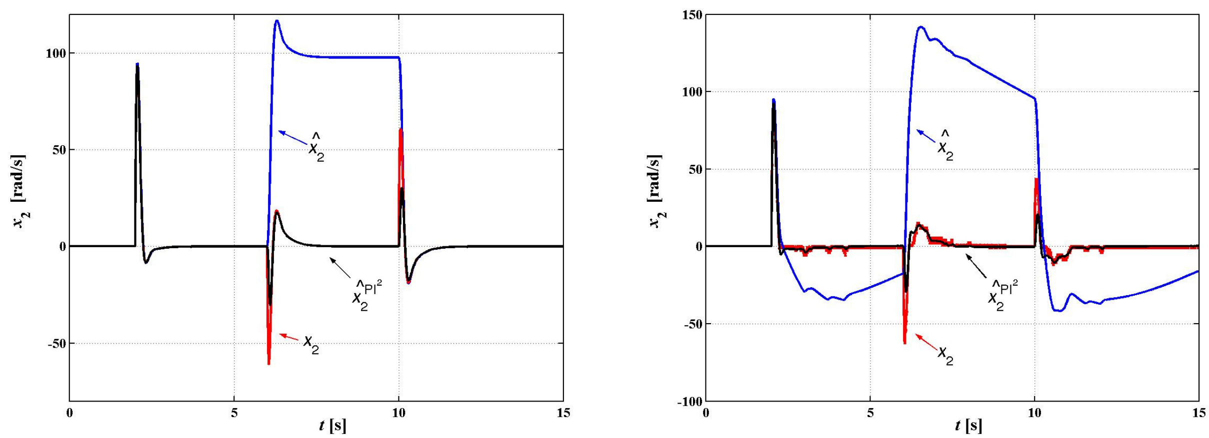
(a) (b)

Fig. 7. Control signal in system with reduced-order observers (a) simulation, (b) experiment.



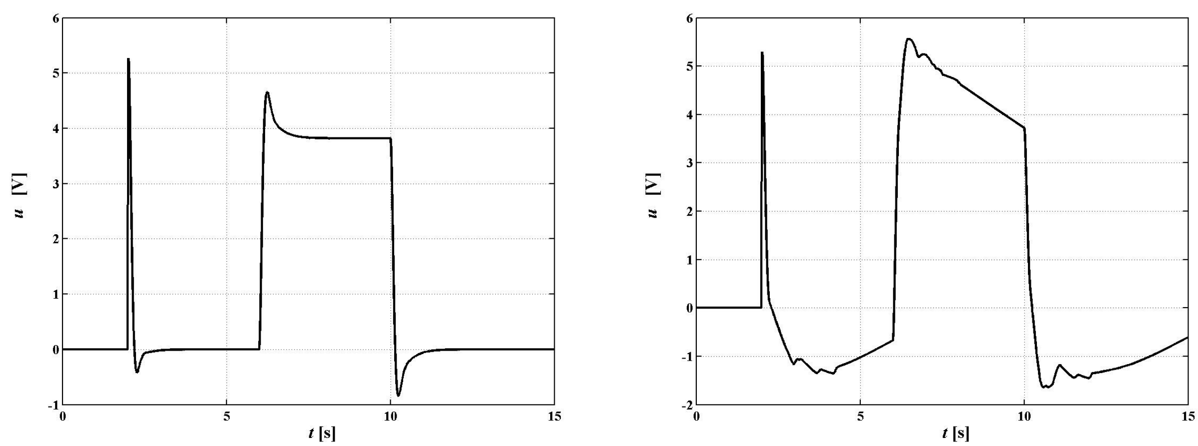
(a) (b)

Fig. 8. True and estimation values of the shaft position using identity observer (15) and PI^2 observer (28) (a) simulation, (b) experiment.



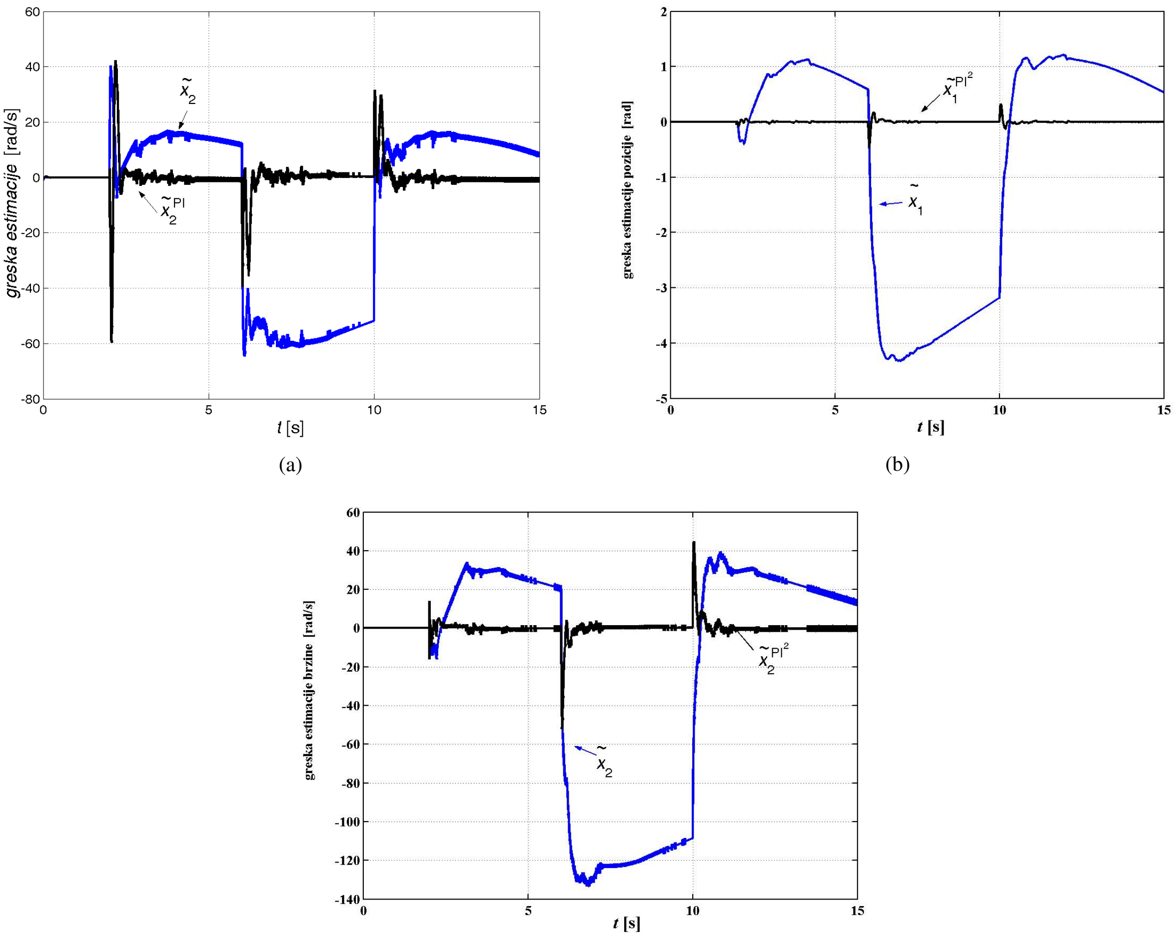
(a) (b)

Fig. 9. True and estimation values of the shaft speed using identity observer (15) and PI^2 observer (28) (a) simulation, (b) experiment.



(a) (b)

Fig. 10. Control signal in system with full-order observers (a) simulation, (b) experiment.



(a) (b)

(c)

Fig. 11. Estimation errors: (a) Estimation error of the shaft speed using reduced-order observer (16) and reduced-order PI observer (22), (b) Estimation error of the shaft position using identity observer (15) and PI^2 observer (28), (c) Estimation error of the shaft speed using identity observer (15) and PI^2 observer (28).

Although a perfect observation paradigm cannot be obtained, the proposed algorithms can effectively control the estimation errors of system states even in the presence of external disturbances.

VI. CONCLUSION

The aim of this paper is to consider the possibility of using the ordinary discrete-time observers full and reduced-order, and their modifications called PI^2 and PI observer for proper speed estimation in the case of the constant or slow varying load torque disturbances. For gain adjustment of observers extended with integral actions the suitable simple procedures are proposed. Simulation results, as well as the real-time experimental results validate the superior performances of the proposed new state observer structures.

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