Self-Oscillating Fluxgate Current Sensor with Pulse Width Modulated Feedback

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Abstract—Several methods, based on the relay and phase-locked loop (PLL) experiment for determining the ultimate frequency, ultimate gain, and Nyquist curve $G_p(s)$, given $\arg(G_p(i\omega))$, are analyzed. To support the practical application of these algorithms in the presence of the higher harmonics, measurement noise, and load disturbances, an adaptive filter (AF) of the band-pass type has been applied. The methods are tested on a large class of typical industrial processes. The SIMULINK realizations of the algorithms are presented.

Index Terms—PID controller, Frequency domain, Conventional relay, PLL.

I. INTRODUCTION

The method of generating stable oscillations for estimating the ultimate frequency $\omega_u$ and ultimate gain $k_u$ of a process $G_p(s)$ is initiated by Ziegler and Nichols [1] in 1942. They proposed to use $\omega_u$ and $k_u$ for dynamic characterization of a process $G_p(s)$ and for the experimental PI/PID controller tuning. Better realization of the Ziegler-Nichols method, proposed by Åström and Hägglund [2] in 1984, is known as the Conventional Relay (CR) experiment. The factors influencing the accuracy of estimation of $\omega_u$ and $k_u$ by applying the CR experiment are: insufficiently filtered out higher harmonics by the process, presence of the load disturbance $d$ and measurement noise $n$. The first drawback of the CR is removed in [3] by application of the Modified Relay (MR).

Further development of methods for determination of $\omega_u$ and $k_u$ applies the principle of the phase locked loop (PLL) [4,5]. The PLL method from [4] removed almost all shortcomings present with the conventional relay experiment. Modifications of the basic relay-based and PLL-based structures of the previously discussed methods are shown in the figures Figs. 1-3.

It will be shown that application of an adaptive filter AF of the band-pass type in Fig. 1. results into a setup denoted by CR-f and allows determination of $\omega_u$ and $k_u$ of process $G_p(s)$ by the conventional relay in the presence of load disturbances and measurement noise.

It will also be shown that application of this filter to MR (Fig. 2.) results into a setup denoted by MR-f and allows, in addition to a reliable determination of $\omega_u$ and $k_u$ of process $G_p(s)$, also a reliable estimation of Nyquist curve $G_p(i\omega)$, given $\theta_{ref}=\arg(G_p(i\omega))$, in the presence of load disturbances and measurement noise.

By applying AF, upgrading of the PLL method from [5] is presented in Fig. 3.
II. ANALYSIS

Modification of the basic schemes, involving conventional (CR) and modified (MR) relay, for determination of \( \omega_u \) and \( k_u \) of process \( G_p(s) \), includes an adaptive filter of the band-pass type. This means that in Fig. 1 and Fig. 2 block AF is realized by applying a cascade of filters defined by transfer function

\[
F(s) = \prod_{n=1}^{4} \frac{\beta_n \omega_n s}{s^2 + \beta_n \omega_n s + \omega_n^2}, \quad \beta_n = 2 \cos(\pi(2m-1)/16) \quad (1)
\]

On the basis of harmonic analysis [6], characteristic equation of the conventional relay including filter (1) is

\[
1 + \frac{4\mu}{\pi A_1} G_p(i \omega_k) F(i \omega_k) = 0 . \quad (2)
\]

Since \( F(i \omega_k) = 1 \), (2) reduces to

\[
1 + \frac{4\mu}{\pi A_1} G_p(i \omega_k) = 0 . \quad (3)
\]

From (2)-(3) it follows that estimates of the critical frequency and critical gain are equal to values \( \omega_u \) and \( k_u \) of process \( G_p(s) \). In (2)-(3)

\[
k_u = \frac{4\mu}{\pi A_1} , \quad (4)
\]

where \( A_1 \) is amplitude of the first harmonic of the Fourier expansion of the output signal of filter (1). Since a cascade of filters is a narrow band-pass filter, all harmonics except the fundamental \( \omega = \omega_0 \) are suppressed. The same applies for the high frequency measurement noise and load disturbances. This removes all shortcomings of the conventional relay experiment [2] mentioned earlier.

The necessary condition for stability of the limit cycle defined by relation (2) is given [7] by:

\[
\frac{\partial \text{Im}(G_p(i \omega) F(i \omega))}{\partial \omega} \bigg|_{\omega = \omega_u} > 0 , \quad (5)
\]

i.e. by

\[
\frac{\partial \text{Im} G_p(i \omega_k)}{\partial \omega_k} \bigg|_{\omega_k = \omega_u} + \frac{2}{k_u \omega_u \beta_0} > 0 , \quad (6)
\]

where \( \beta_0 = 1/\sum_{m=1}^{4} 1/\beta_m \) and \( k_u \) from (4) satisfies condition \( k_u > 0 \).

Then, under condition that the solution defined by relation (3) satisfies the necessary condition for stability of the limit cycle, condition (6) is satisfied for the limit cycle defined by relation (2).

The structure shown in Fig. 3, in addition to the good properties as regards determination of \( \omega_u \) and \( k_u \) and estimation of Nyquist curve \( G_p(i \omega) \) for a given \( \theta_{\text{ref}} \), has the following shortcomings. Adjustment of parameter \( k_{\text{PLL}} \) in regulator \( R_{\text{PLL}} \) is dependent on the tested process \( G_p(s) \). If the tested process is unstable, the structure of PLL of Fig. 3 is not applicable for any value of parameter \( k_{\text{PLL}} \). Generalization of this concept involving PLL proposed in [4] is applicable also to unstable processes. However, in both approaches [4,5], in the case of multiple solution of equation

\[
\arg \{ G_p(i \omega) \} = \theta_{\text{ref}} , \quad (7)
\]

as well as for the solutions having \( \arg \{ G_p(i \omega) \} > 2\pi \), the PLL concept requires a priori knowledge of the frequency of interest. This also means the following: if \( \omega_u \) is being determined and one starts from an initial guess which is close to another solution of equation (7) for \( \theta_{\text{ref}} = -\pi \), the ultimate frequency of the considered process \( G_p(s) \) will not be determined. In this sense the concepts based on the relay principle have the advantage since they give unique solution, [8].

Figs. 4.-5. show the schemes involving PLL for determination of \( \omega_u \) and \( k_u \) of \( G_p(i \omega) \) for given values \( \theta_{\text{ref}} = \arg \{ G_p(i \omega) \} \) of process \( G_p(s) \) without breaking the loop of the controller in operation. Compared to the systems considered so far, in this approach regulator \( C(s) \) is not disconnected, as done in Figs. 1.-3.

Improvement of the solution from [4], proposed in the present paper, consists of applying AF filter to avoid problems associated with measurement noise and load disturbances. This considerably simplifies the implementation and adequate application of the PLL concept to a closed-loop system with regulator \( C(s) \) in operation with process \( G_p(s) \). The solution proposed in [4] uses Kalman filter for eliminating influence of measurement noise and averaging technique for eliminating influence of load disturbances.

Application of the PLL concept to a closed-loop control system in Fig. 4, with inserted filter (1), follows directly from relations

\[
\arg \left\{ \frac{Y_p(i \omega)}{U_{\text{ref}}(i \omega)} \right\} - \arg \left\{ \frac{U_{\text{ref}}(i \omega)}{U_{\text{ref}}(i \omega)} \right\} = \arg \left\{ G_p(i \omega) \right\} \quad (8)
\]

and

\[
\left| \frac{Y_p(i \omega)}{U_{\text{ref}}(i \omega)} \right| + \left| \frac{U_{\text{ref}}(i \omega)}{U_{\text{ref}}(i \omega)} \right| = \left| G_p(i \omega) \right| . \quad (9)
\]

Fig. 4. Application of the PLL estimation of \( G_p(i \omega) \) without braking the control loop in operation.

On the basis of (9), critical gain \( k_u \) is directly obtained from the ratio of fundamental harmonics of Fourier expansions of output signals \( u_\text{ref} \) and \( y_p \) from AF in Fig. 5,

\[
k_u = A_{\text{ref}} / A_y . \quad (10)
\]
III. SIMULATION RESULTS

A comparison of the previously analyzed methods is presented in Table 1 by taking fourteen representatives of the typical dynamic characteristics of industrial processes having transfer functions:

\[
G_p(s) = \frac{1}{(s+1)^2}, \quad G_{p1}(s) = 1/\left(0.7s + 1\right), \quad G_{p2}(s) = \frac{12.8e^{-s}}{16.8s + 1},
\]

\[
G_{p3}(s) = \frac{e^{-s}}{(s+1)^2}, \quad G_{p4}(s) = \frac{10}{(s+1)(0.7s + 1)(0.1s + 1)}, \quad G_{p5}(s) = e^{-s},
\]

\[
G_{p6}(s) = \frac{1 - 2s}{(s+1)^2}, \quad G_{p7}(s) = \frac{1}{\cosh 2s}, \quad G_{p8}(s) = \frac{1}{s(s+1)^2},
\]

\[
G_{p9}(s) = \frac{9}{(s+1)(s^2 + 2s + 9)}, \quad G_{p10}(s) = \frac{(s + 0.2)e^{-0.5s}}{s^2 + s + 1},
\]

\[
G_{p11}(s) = \frac{4e^{-2s}}{4s - 1}, \quad G_{p12}(s) = \frac{e^{-s}}{s}, \quad G_{p13}(s) = \frac{1.078e^{-10s}}{s^2 + 0.14s + 0.49}.
\]

The Simulink scheme shown in Fig. 6. is used, for determination of the frequency \( \omega_u \) by the relay methods.

As demonstrated in Table 1, all methods give good \( \omega_u \) and \( k_u \) for the processes without the load disturbances and measurement noise. Efficiency of the MPLL-f for the unstable plant \( G_{p12}(s) \) should be particularly emphasized.

Differences between these methods in the presence of the measurement noise and load disturbances, due to space limitation, will be illustrated using the process \( G_{p1}(s) \). When method MPLL-f was applied, PID regulator based on \([9,10]\) was used.

![Simulink scheme for determination of argument θ from interval 0 < θ < 2π, where K1=1/β and AF/1 is single cell of AF filter for adopted β = √2.](image)

![Simulink scheme for determination of the frequency ω_u by the relay methods.](image)

![Simulink scheme for determination of the frequency ω_u at the output of AF filter.](image)

![Simulink scheme of AF/1 a single cell of AF filter.](image)

![Modified PLL estimator (MPLL-f), with adaptive band-pass filter AF and low-pass filter NF.](image)
Fig. 9. Dynamic characteristics of estimated critical values for process \( G_p(t) \) in the presence of load disturbance \( d=2\mu \) at instant \( t_0=200 \), applying methods CR-f, MR-f, PLL-f, and MPLL-f a) critical frequency \( \omega_u \) and b) critical gain \( k_u \). Time constant in the NF filter is \( T_{nf}=1 \).

However, it should be emphasized that the method MPLL-f is independent of the applied regulator. It is applicable to processes in the loop with the badly tuned PI or PID controller. In this case, on-line monitoring of parameters \( \omega_u \) and \( k_u \) and a reliable estimate of Nyquist curve \( G_p(i\omega) \), given values \( \theta_{ref}=\text{arg}\{G_p(i\omega)\} \), makes possible to perform fine tuning without interrupting the control loops in operation. In other words, an on-line adaptation is possible based on estimation offered by MPLL-f and by using PID optimization form [10] or tuning from [11].

Fig. 10. Time diagram of the measured signal corrupted by noise of “random number” type of variance 0.01 and time diagram of the filtered signal \( y_f \) at the output of AF filter.

Fig. 11. Dynamic characteristics of the estimated critical values for process \( G_p(t) \) in the presence of measurement noise of “random number” type of variance 0.01, applying methods CR-f, MR-f, PLL-f, and MPLL-f a) critical frequency \( \omega_u \) and b) critical gain \( k_u \). In the NF filter \( T_{nf}=1 \).

Fig. 12. Estimates of Nyquist curve \( G_p(i\omega) \), given values \( \theta_{ref}=\text{arg}\{G_p(i\omega)\} = -\pi + \theta_\phi = \pi/2, \pi/3, \pi/6, 0, -\pi/6, -\pi/3 \), by applying MR-f, PLL-f, and MPLL-f.

Fig. 13. Estimates of Nyquist curve \( G_p(i\omega) \), given values \( \theta_{ref}=\text{arg}\{G_p(i\omega)\} = -\pi + \theta_\phi = \pi/2, \pi/3, \pi/6, 0, -\pi/6, -\pi/3 \), by applying MR-f, PLL-f, and MPLL-f.

IV. Conclusion

The proposed modifications of the relay-based and PLL-based estimators, denoted by CR-f, MF-f, PLL-f and MPLL-f, showed adequate for a wide class of typical industrial processes. Highly accurate estimates of \( \omega_u \) and \( k_u \), and estimate of Nyquist curve \( G_p(i\omega) \), given values \( \theta_{ref}=\text{arg}\{G_p(i\omega)\} \), are obtained in the presence of measurement noise and load disturbances.
In addition, this comparative analysis confirmed that by applying AF filter makes the relay-based estimators (CR-f, MR-f) effective in the presence of the load disturbances and measurement noise.

Finally, the proposed MPLL-f modification of the PLL method from [4], significantly simplifies determination of $\omega_u$ and $k_u$, and estimation of Nyquist curve $G_p(i\omega)$ for given values $\theta_{ref}=\arg\{G_p(i\omega)\}$, without breaking the control loops in operation. This makes possible fine tuning [11] of a controller while in operation.

REFERENCES