Self-Oscillating Fluxgate Current Sensor with Pulse Width Modulated Feedback

Tomislav B. Šekara and Miroslav R. Mataušek

***Abstract*—Several methods, based on the relay and phase-locked loop (PLL) experiment for determining the ultimate frequency, ultimate gain, and Nyquist curve Gp(iω), given arg(Gp(iω)), are analyzed. To support the practical application of these algorithms in the presence of the higher harmonics, measurement noise, and load disturbances, an adaptive filter (AF) of the band-pass type has been applied. The methods are tested on a large class of typical industrial processes. The SIMULINK realizations of the algorithms are presented.**

***Index Terms*—PID controller, Frequency domain, Conventional relay, PLL.**

I. Introduction

THE method of generating stable oscillations for estimating the ultimate frequency ω_u and ultimate gain k_u of a process $G_p(s)$ is initiated by Ziegler and Nichols [1] in 1942. They proposed to use ω_u and k_u for dynamic characterization of a process $G_p(s)$ and for the experimental PI/PID controller tuning. Better realization of the Ziegler-Nichols method, proposed by Åström and Hägglund [2] in 1984, is known as the Conventional Relay (CR) experiment. The factors influencing the accuracy of estimation of ω_u and k_u by applying the CR experiment are: insufficiently filtered out higher harmonics by the process, presence of the load disturbance d and measurement noise n. The first drawback of the CR is removed in [3] by application of the Modified Relay (MR).

Further development of methods for determination of ω_u and k_u applies the principle of the phase locked loop (PLL) [4,5]. The PLL method from [4] removed almost all shortcomings present with the conventional relay experiment. Modifications of the basic relay-based and PLL-based structures of the previously discussed methods are shown in the figures Figs. 1-3.

It will be shown that application of an adaptive filter AF of the band-pass type in Fig. 1. results into a setup denoted by CR-f and allows determination of ω_u and k_u of process $G_p(s)$ by the conventional relay in the presence of load disturbances and measurement noise.

It will also be shown that application of this filter to MR (Fig. 2.) results into a setup denoted by MR-f and allows, in addition to a reliable determination of ω_u and k_u of process $G_p(s)$, also a reliable estimation of Nyquist curve $G_p(i\omega)$, given $\theta_{ref}=\arg\{G_p(i\omega)\}$, in the presence of load disturbances and measurement noise,.

By applying AF, upgrading of the PLL method from [5] is presented in Fig. 3.

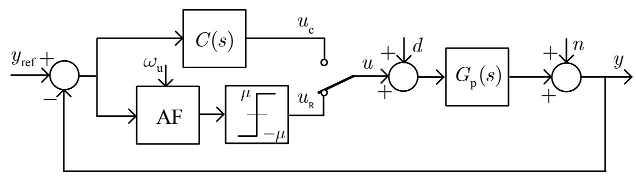

Fig. 1. Basic scheme of the conventional relay for determination of ω_u and k_u of process $G_p(s)$, denoted by CR for AF≡1. Modification CR-f is obtained for AF≠1.

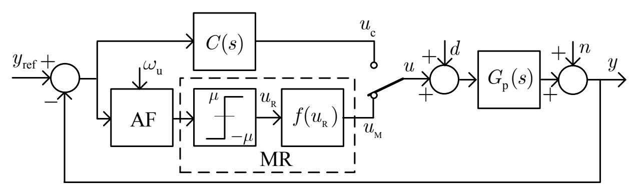

Fig. 2. Basic scheme of the modified relay for determination of ω_u and k_u of process $G_p(s)$, denoted by MR for AF≡1. Modification MR-f is obtained for AF≠1.

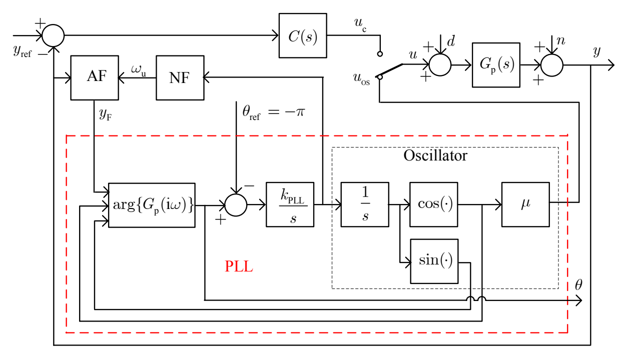

Fig. 3. Modified PLL-f scheme for determination of ω_u and k_u of process $G_p(s)$ involves an adaptive filter AF of the band-pass type and a low-pass filter NF. In original PLL scheme from [5] one cell of AF filter, shown in Fig. 7, is used, with NF≡1.

Finally, by applying AF, upgrading of the PLL method from [4] for determination of ω_u, k_u, and estimation of Nyquist curve $G_p(i\omega)$, given θ_{ref}, will be proposed, denoted by MPLL-f. Contrary to methods presented in Figs. 1-3, in the MPLL-f method breaking the loop of controller $C(s)$ in operation is avoided. Introduction of a low-pass filter NF makes the MPLL-f approach robust for determination of ω_u, k_u, and estimation of Nyquist curve $G_p(i\omega)$, given θ_{ref}.

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II. ANALYSIS

Modification of the basic schemes, involving conventional (CR) and modified (MR) relay, for determination of ω_u and k_u of process $G_p(s)$, includes an adaptive filter of the band-pass type. This means that in Fig. 1 and Fig. 2 block AF is realized by applying a cascade of filters defined by transfer function

$$F(s)=\prod_{m=1}^{4}\frac{\beta_m\omega_u s}{s^2+\beta_m\omega_u s+\omega_u^2},\ \beta_m=2\cos(\pi(2m-1)/16) \quad (1)$$

On the basis of harmonic analysis [6], characteristic equation of the conventional relay including filter (1) is

$$1+\frac{4\mu}{\pi A_1}G_p(i\omega_u)F(i\omega_u)=0. \quad (2)$$

Since $F(i\omega_u)=1$, (2) reduces to

$$1+\frac{4\mu}{\pi A_1}G_p(i\omega_u)=0. \quad (3)$$

From (2)-(3) it follows that estimates of the critical frequency and critical gain are equal to values ω_u and k_u of process $G_p(s)$. In (2)-(3)

$$k_u=\frac{4\mu}{\pi A_1}, \quad (4)$$

where A_1 is amplitude of the first harmonic of the Fourier expansion of the output signal of filter (1). Since a cascade of filters is a narrow band-pass filter, all harmonics except the fundamental $\omega=\omega_u$ are suppressed. The same applies for the high frequency measurement noise and load disturbances. This removes all shortcomings of the conventional relay experiment [2] mentioned earlier.

The necessary condition for stability of the limit cycle defined by relation (2) is given [7] by:

$$\left.\frac{\partial\,\mathrm{Im}(G_p(i\omega)F(i\omega))}{\partial\omega}\right|_{\omega=\omega_u}>0, \quad (5)$$

i.e. by

$$\left.\frac{\partial\,\mathrm{Im}\,G_p(i\omega)}{\partial\omega}\right|_{\omega=\omega_u}+\frac{2}{k_u\omega_u\beta_0}>0, \quad (6)$$

where $\beta_0=1/\sum_{m=1}^{4}1/\beta_m$ and k_u from (4) satisfies condition $k_u>0$. Then, under condition that the solution defined by relation (3) satisfies the necessary condition for stability of the limit cycle, condition (6) is satisfied for the limit cycle defined by relation (2).

The structure shown in Fig. 3, in addition to the good properties as regards determination of ω_u and k_u and estimation of Nyquist curve $G_p(i\omega)$ for a given θ_{ref}, has the following shortcomings. Adjustment of parameter k_{PLL} in regulator k_{PLL}/s is dependent on the tested process $G_p(s)$. If the tested process is unstable, the structure of PLL of Fig. 3 is not applicable for any value of parameter k_{PLL}. Generalization of this concept involving PLL proposed in [4] is applicable also to unstable processes. However, in both approaches [4,5], in the case of multiple solution of equation

$$\arg\{G_p(i\omega)\}=\theta_{ref}, \quad (7)$$

as well as for the solutions having $\arg\{G_p(i\omega)\}>2\pi$, the PLL concept requires a priori knowledge of the frequency of interest. This also means the following: if ω_u is being determined and one starts from an initial guess which is close to another solution of equation (7) for $\theta_{ref}=-\pi$, the ultimate frequency of the considered process $G_p(s)$ will not be determined. In this sense the concepts based on the relay principle have the advantage since they give unique solution, [8].

Figs. 4.-5. show the schemes involving PLL for determination of ω_u and k_u, and $G_p(i\omega)$ for given values $\theta_{ref}=\arg\{G_p(i\omega)\}$, of process $G_p(s)$ without breaking the loop of the controller in operation. Compared to the systems considered so far, in this approach regulator $C(s)$ is not disconnected, as done in Figs. 1.-3.

Improvement of the solution from [4], proposed in the present paper, consists of applying AF filter to avoid problems associated with measurement noise and load disturbances. This considerably simplifies the implementation and adequate application of the PLL concept to a closed-loop system with regulator $C(s)$ in operation with process $G_p(s)$. The solution proposed in [4] uses Kalman filter for eliminating influence of measurement noise and averaging technique for eliminating influence of load disturbances.

Application of the PLL concept to a closed-loop control system in Fig. 4, with inserted filter (1), follows directly from relations

$$\arg\left\{\frac{Y_F(i\omega)}{U_{OS}(i\omega)}\right\}-\arg\left\{\frac{U_{cF}(i\omega)}{U_{OS}(i\omega)}\right\}=\arg\{G_p(i\omega)\} \quad (8)$$

and

$$\left|\frac{Y_F(i\omega)}{U_{OS}(i\omega)}\right|\div\left|\frac{U_{cF}(i\omega)}{U_{OS}(i\omega)}\right|=\left|G_p(i\omega)\right|. \quad (9)$$

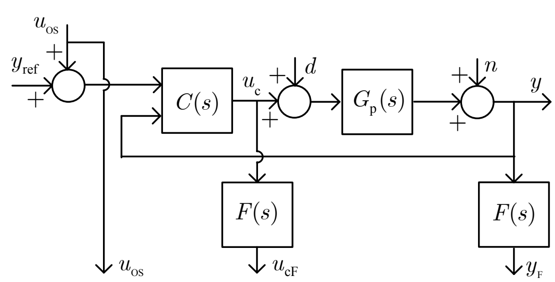

Fig. 4. Application of the PLL estimation of $G_p(i\omega)$ without braking the control loop in operation.

On the basis of (9), critical gain k_u is directly obtained from the ratio of fundamental harmonics of Fourier expansions of output signals u_{cF} and y_F from AF in Fig. 5,

$$k_u=A_{1cF}/A_{1F}. \quad (10)$$

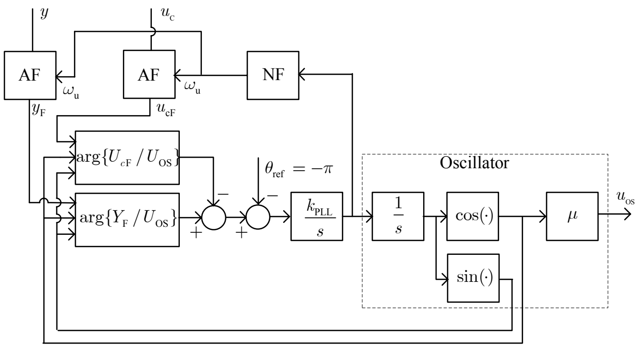

Fig. 5. Modified PLL estimator (MPLL-f), with adaptive band-pass filter AF and low-pass filter NF.

III. Simulation Results

A comparison of the previously analyzed methods is presented in Table 1 by taking fourteen representatives of the typical dynamic characteristics of industrial processes having transfer functions:

$$G_{p1}(s)=\frac{1}{(s+1)^4},\ G_{p2}(s)=1/\prod_{k=0}^{3}(0.7^k s+1),\ G_{p3}(s)=\frac{12.8e^{-s}}{16.8s+1},$$

$$G_{p4}(s)=\frac{e^{-5s}}{(s+1)^3},\ G_{p5}(s)=\frac{10}{(s+1)(0.7s+1)(0.1s+1)},\ G_{p6}(s)=e^{-\sqrt{s}},$$

$$G_{p7}(s)=\frac{1-2s}{(s+1)^3},\quad G_{p8}(s)=\frac{1}{\cosh\sqrt{2s}},\ G_{p9}(s)=\frac{1}{s(s+1)^3},$$

$$G_{p10}(s)=\frac{9}{(s+1)(s^2+2s+9)},\ G_{p11}=\frac{(s+0.2)e^{-0.5s}}{s^2+s+1},$$

$$G_{p12}(s)=\frac{4e^{-2s}}{4s-1},\ G_{p13}(s)=\frac{e^{-s}}{s},\ G_{p14}(s)=\frac{1.078e^{-10s}}{s^2+0.14s+0.49}.$$

The Simulink scheme shown in Fig. 6. is used, for determination of the frequency ω_u by the relay methods.

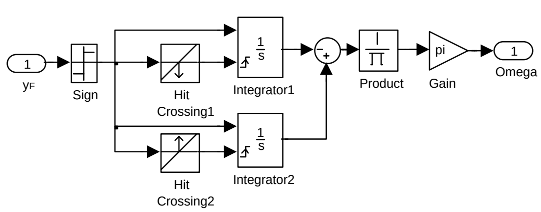

Fig. 6. Simulink scheme for determination of the frequency ω_u at the output of AF filter.

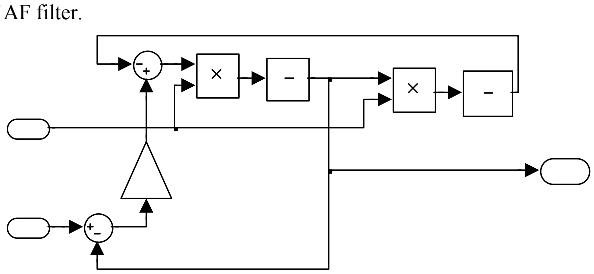

Fig. 7. Simulink scheme of AF/1 a single cell of AF filter.

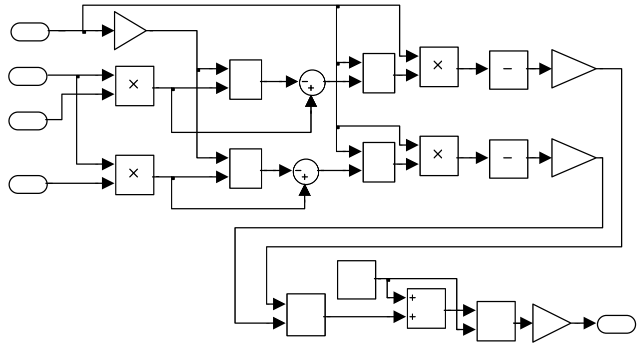

Fig. 8. Simulink scheme for determination of argument θ from interval 0 < θ < 2π, where K1=1/β and AF/1 is single cell of AF filter for adopted β = √2.

TABLE 1
COMPARISON OF METHODS CR, MR, CR-F, MR-F, PLL-F AND MPLL-F FOR DETERMINATION OF ΩU AND KU OF THE REPRESENTATIVE TYPICAL INDUSTRIAL PROCESSES GPJ(S), J=1,...14.

$G_{pj}(s)$ ω_{uj}/k_{uj}	Exact	CR	MR	CR-f	MR-f	PLL-f	MPLL-f
ω_{u1}	1.000	0.993	1.000	1.000	1.000	1.000	1.000
k_{u1}	4.000	3.945	4.000	4.000	4.000	4.000	4.000
ω_{u2}	1.707	1.693	1.707	1.707	1.707	1.707	1.707
k_{u2}	4.659	4.581	4.659	4.659	4.659	4.659	4.659
ω_{u3}	1.608	1.616	-	1.606	1.608	1.608	1.608
k_{u3}	2.112	2.127	-	2.109	2.112	2.112	2.112
ω_{u4}	0.400	0.410	0.400	0.400	0.400	0.400	0.400
k_{u4}	1.249	1.262	1.249	1.249	1.249	1.249	1.249
ω_{u5}	3.684	3.613	3.684	3.685	3.684	3.684	3.684
k_{u5}	1.311	1.258	1.311	1.312	1.311	1.311	1.311
ω_{u6}	19.74	19.31	19.74	19.73	19.74	19.74	19.74
k_{u6}	23.14	22.43	23.14	23.13	23.14	23.14	23.14
ω_{u7}	0.845	0.788	0.845	0.845	0.845	0.845	0.845
k_{u7}	1.143	1.105	1.143	1.143	1.143	1.143	1.143
ω_{u8}	9.869	9.679	9.869	9.867	9.869	9.869	9.869
k_{u8}	11.59	11.26	11.59	11.59	11.59	11.59	11.59
ω_{u9}	0.577	0.567	-	0.577	0.577	0.577	0.577
k_{u9}	0.889	0.860	-	0.889	0.889	0.889	0.889
ω_{u10}	3.317	3.306	3.317	3.317	3.317	3.317	3.317
k_{u10}	2.667	2.643	2.667	2.667	2.667	2.667	2.667
ω_{u11}	3.613	3.700	3.613	3.610	3.613	3.613	3.613
k_{u11}	3.478	3.568	3.478	3.475	3.478	3.478	3.478
ω_{u12}	0.583	0.508	-	-	-	-	0.583
k_{u12}	0.634	0.566	-	-	-	-	0.634
ω_{u13}	1.571	1.571	-	1.571	1.571	1.571	1.571
k_{u13}	1.571	1.571	-	1.571	1.571	1.571	1.571
ω_{u14}	0.737	0.737	0.737	0.737	0.737	0.737	0.737
k_{u14}	0.108	0.108	0.108	0.108	0.108	0.108	0.108

As demonstrated in Table 1, all methods give good ω_u and k_u for the processes without the load disturbances and measurement noise. Efficiency of the MPLL-f for the unstable plant $G_{p12}(s)$ should be particularly emphasized.

Differences between these methods in the presence of the measurement noise and load disturbances, due to space limitation, will be illustrated using the process Gp1(s). When method MPLL-f was applied, PID regulator based on [9,10] was used.

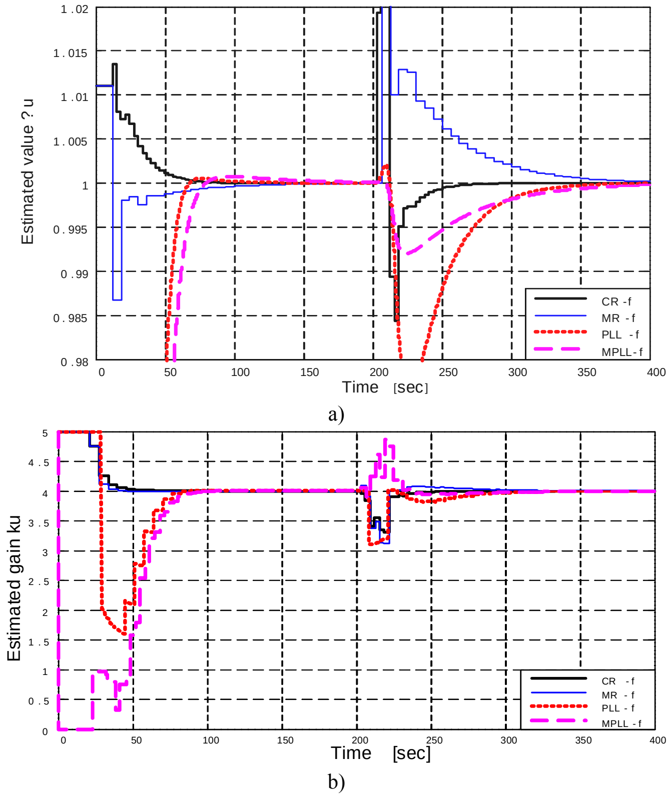

Fig. 9. Dynamic characteristics of estimated critical values for process $G_{P1}(s)$ in the presence of load disturbance d=2μ at instant t_0=200, applying methods CR-f, MR-f, PLL-f, and MPLL-f a) critical frequency ω_u and b) critical gain k_u. Time constant in the NF filter is T_{nf}=1.

However, it should be emphasized that the method MPLL-f is independent of the applied regulator. It is applicable to processes in the loop with the badly tuned PI or PID controller. In this case, on-line monitoring of parameters ω_u and k_u and a reliable estimate of Nyquist curve $G_p(i\omega)$, given values θ_{ref} =arg{$G_p(i\omega)$}, makes possible to perform fine tuning without interrupting the control loops in operation. In other words, an on-line adaptation is possible based on estimation offered by MPLL-f and by using PID optimization form [10] or tuning from [11].

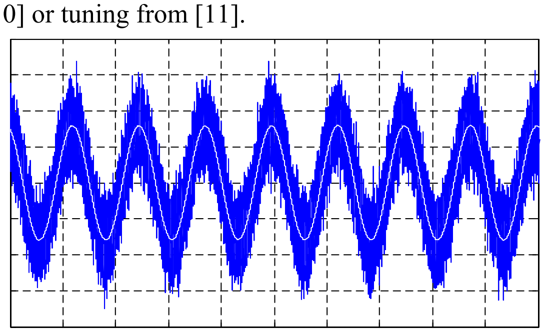

Fig. 10. Time diagram of the measured signal corrupted by noise of „random number" type of variance 0.01 and time diagram of the filtered signal y_f at the output of AF filter.

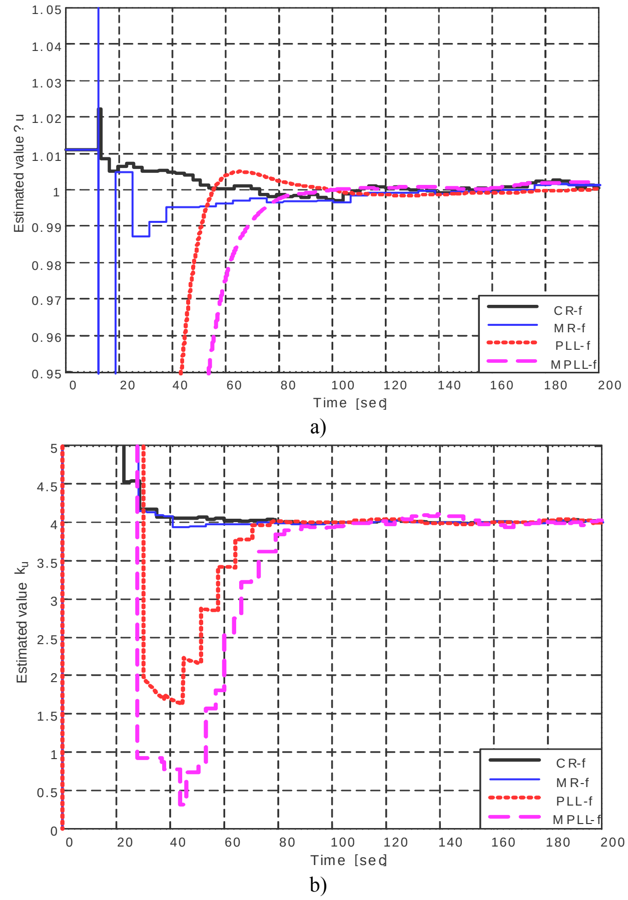

Fig. 11. Dynamic characteristics of the estimated critical values for process $G_{p1}(s)$ in the presence of measurement noise of "random number" type of variance 0.01, applying methods CR-f, MR-f, PLL-f, and MPLL-f a) critical frequency ω_u and b) critical gain k_u. In the NF filter T_{nf}=1.

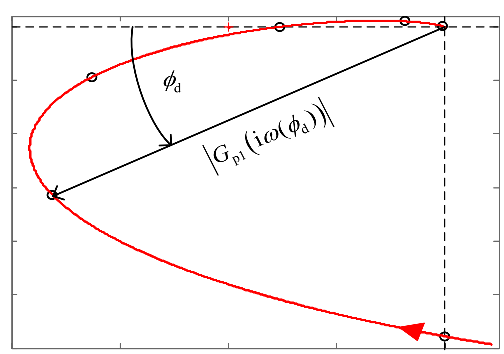

Fig. 12. Estimates of Nyquist curve $G_{p5}(i\omega)$, given values θ_{ref}=arg{$G_p(i\omega)$}= -π + ϕ_d, ϕ_d=π/2, π/3, π/6, 0, -π/6, -π/3, by applying MR-f, PLL-f, and MPLL-f.

IV. Conclusion

The proposed modifications of the relay-based and PLL-based estimators, denoted by CR-f, MF-f, PLL-f and MPLL-f, showed adequate for a wide class of typical industrial processes. Highly accurate estimates of ω_u and k_u, and estimate of Nyquist curve $G_p(i\omega)$, given values θ_{ref}=arg{$G_p(i\omega)$}, are obtained in the presence of measurement noise and load disturbances.

In addition, this comparative analysis confirmed that by applying AF filter makes the relay-based estimators (CR-f, MR-f) effective in the presence of the load disturbances and measurement noise.

Finally, the proposed MPLL-f modification of the PLL method from [4], significantly simplifies determination of ω_u and k_u, and estimation of Nyquist curve $G_p(i\omega)$ for given values $\theta_{ref}=\arg\{G_p(i\omega)\}$, without breaking the control loops in operation. This makes possible fine tuning [11] of a controller while in operation.

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