Mutual Inductance Variation Influence on Induction Motor IFOC Drive

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***Abstract*—This paper describes a solution to the mutual inductance variation problem in the Indirect Field Oriented (IFOC) controlled induction motor. Initially, the influence of the variation on the produced flux, torque and torque per ampere is analyzed and experimentally verified. After that, a simple model modification is proposed and discussed. Finally, the solution is implemented on an AC motor-drive and verified experimentally by comparing the torque step response before the modification, and after it.**

***Index Terms*—Induction motor, IFOC, Parameter variation, Nonlinearities, Saturation, Mutual inductance variation.**

I. Introduction

IN the contemporary high performance AC motor drives, the most widely used scheme is an indirect field oriented control (IFOC) with current controllers in *d-q* reference frame tied to the rotor flux vector. In this type of drives, a mismatch between machine parameters used in the controller and the actual machine parameters, which occurs due to changes in temperature or saturation, results in the following [1]:

- The flux level is not properly maintained, and the rotor flux amplitude is not equal to the expected value.
- The resulting steady state torque is not equal to the commanded values.
- The torque response is not instantaneous.
- The correlation between i_{qs} and torque, as well as the correlation between i_{ds} and flux is not linear anymore.

These effects are widely documented in literature [2,3]. In the indirect field oriented control schemes with current controllers in *d-q* reference frame tied to the rotor flux vector, the parameter of greatest interest is the rotor time constant τ_r, $\tau_r = L_r / R_r = (M + L_{\sigma r})/R_r$, where L_r, R_r and $L_{\sigma r}$ are rotor inductance, rotor resistance and rotor leakage inductance, respectively. M denotes mutual inductance between stator and rotor.

A wrongly set parameter in the model yields the differrence between the commanded and obtained values of the currents even if the current controllers are ideal. Both, the torque-producing (i_{qs}) and the flux-producing (i_{ds}) currents are affected.

This paper primarily deals with the influence of the mutual inductance mismatch (difference between the value of parameter M used in the controller and the real value of M in the machine) due to saturation and experimentally verifies a simple mathematical-model modification that reduces the above mentioned influence on the drive performance.

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II. Mathematical Model

A model of an induction machine dynamics (assuming linear magnetic), in the *d-q* frame oriented so that the *d*-axis is aligned with the total rotor flux vector [1], is shown in Fig. 1.

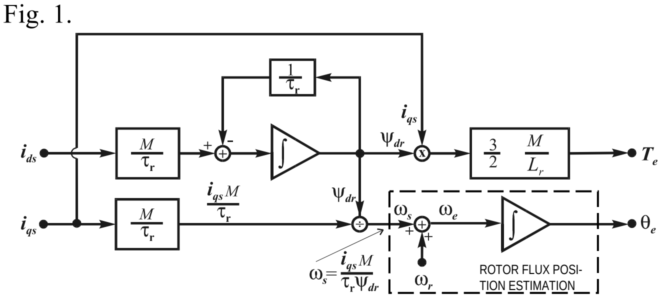

Fig. 1. Induction machine dynamics model in the *d-q* frame.

In Fig. 1, i_{ds} denotes the stator current *d*-component (flux producing current). This current component affects the rotor flux ψ_{dr} only. i_{qs} is the stator current *q*-component which controls the electrical torque T_e exclusively (has no influence on the rotor flux ψ_{dr}). ω_s is the angular slipping frequency, and ω_e and ω_r are the excitation (stator), and the rotor angular frequency, respectively. θ_e is current rotor flux angle, and p_p is the number of motor pole pairs.

This model assumes a correct orientation of the *d-q* reference frame. To get the correct orientation, the controller that controls *the* drive must have the correct information about the current rotor flux angle θ_e because this angle is used in the transformation of the measured currents. The result of the transformation is the value of the stator current components i_{ds} and i_{qs}, used as the input in the model. For this reason (to get current θ_e) the controller contains a software copy of the model. The software model runs in parallel with the machine following the dynamics of all variables.

The difference between the real model in the machine and the software model is that the latter has the reference currents i_{ds}^* and i_{qs}^* as the model input (instead of i_{ds} and i_{qs}).

The torque producing current reference i_{qs}^* (q-component) controls the torque while the d-component reference (or flux-producing current) i_{ds}^* controls the amplitude of the rotor flux. By means of i_{qs}^* the controller calculates the angular slip frequency as:

$$\omega_s^* = \frac{i_{qs}^* M^*}{\tau_r^* \psi_{dr}^*},$$

where τ_r^* presents the rotor time constant value used as a parameter in the controller. The rotor time constant is defined as $\tau_r = (M+L_{\sigma r})/R_r$. Stars in superscript for motor parameters identify the values used in mathematical model (in the controller) and are not necessarily equal to actual parameters in the machine.

The integral of ω_s^* presents the "slip contribution" to the rotor flux position, and is used to estimate (by adding to measured rotor position) current rotor flux position (θ_e^*), essential for coordinate transformations. Since the controller calculates ω_s^* using wrong parameter values, the estimated angle θ_e^* is not equal to the real rotor flux position (angle θ_e) in the machine. Unfortunately, the controller uses this, wrongly estimated, angle θ_e^* to rotate (transform from the stationary to rotated frame) the measured stator currents $i_{\alpha s}$ and $i_{\beta s}$. The transformation (rotation) is necessary to get the d and q component of the stator current because the current controllers work in the synchronous rotating d-q frame.

Critical parameters for the correct θ_e estimation are the rotor resistance R_r and the mutual inductance M. The leakage inductance $L_{\sigma r}$ which is also included in τ_r expression, is usually much smaller than M, appears always in sum with much larger M, and does not vary too much.

Rotor resistance varies a lot with the machine heating, and the mutual inductance M depends on current i_{ds} (or, more precisely, on magnetizing current, i_m, which is equal to i_{ds} in steady state). This paper discusses only the influence of mutual inductance variations. The rotor resistance R_r is considered as a known constant in the simulations, and the experiments are carried out in such a way that it could be treated constant as well.

III. Saturation Effects

Fig. 2. shows the magnetizing curve of the experimental motor and the shape of variation of the mutual inductance with the d-component of the stator current I_{ds} (which is equal to the magnetizing current i_m in steady state [5,6,8]). I_{ds} is a symbol for i_{ds} in steady state.

The diagram shows that the parameter M, for I_{ds} larger than nominal (I_{dsn} = 3,59 A), changes the value considerably. The value in the linear part is almost 2.5 times larger than the value of M at I_{ds} = 12 A.

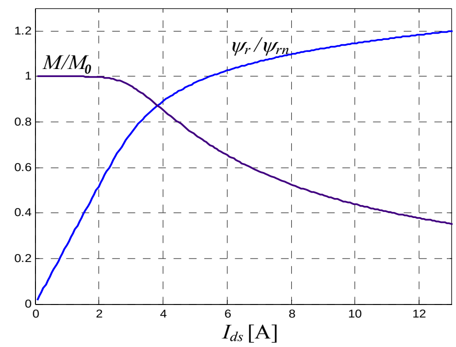

Fig. 2. Experimentally obtained magnetizing curve ψ_r / ψ_{rn} and relative mutual inductance M/M_0 for the experimental machine. M_0 is the mutual inductance in the linear part (163,7mH) and ψ_{rn} is the nominal motor flux (0,59Wb).

Even if the reference value I_{ds}^* never exceeds the nominal, the actual I_{ds} can be several times larger than expected if the parameter R_r^* used in the controller differs from R_r in the machine [7]. For these reasons, the variation of M cannot be neglected.

The importance of taking this variation into account can be seen from the maximum available torque with limited stator current amplitude ("Torque per Ampere" characteristic [8,9]).

Fig. 3. shows the maximum available electrical torque T_e as a function of the slipping frequency f_s with the limited current capacity of the inverter (which is certainly the case). The diagram is obtained by simulation of the experimental machine, described in chapter 5.

The maximum allowed inverter current (which is proportional to $|I_s|$) was first limited to 10A. The reference I_{ds}^* was varied from 0.25 I_{dsn} (0.9A) to $|I_s|$ in 0.05A increments. The entire remaining portion of the inverter current capacity (from 10A) was used for I_{qs}^*. For instance, in the point where I_{ds}^* = 6A for I_{qs}^* was left 8A ($\sqrt{10^2 - 6^2}$). For all obtained I_{ds}^* – I_{qs}^* pairs, the values of slipping frequencies f_s ($f_s = \omega_s / 2\pi$) and electrical torque T_e was calculated using the model given in Fig. 1. The produced electrical torque T_e was shown as a function of f_s. The simulations were repeated for $|I_s|$ = 8A, then for 5A and finally, for 3A.

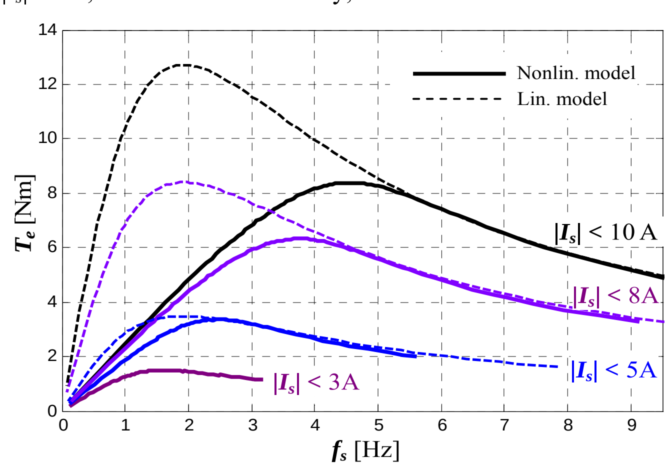

Fig. 3. Produced electrical torque as a function of slip speed for limited stator current magnitude $|I_s|$ using linear and nonlinear machine model. Simulation result for the experimental machine.

In Fig. 3., solid line presents the simulation result with the model that takes into account the variation of M as a function of I_{ds} (as shown in Fig. 2), and dashed line presents the result with the linear model ($M = M_0 =$ const).

It is obvious that the maximum achievable torque is considerably different. Moreover, using the linear model, maximum moment is always achieved for $\omega_{s0} = 1/\tau_r$ [7] independently of the limited $|I_s|$ value. In the nonlinear model, the angular slip frequency that produces the maximum torque is different from ω_{s0}, and the difference is larger for larger stator currents.

IV. Saturated Machine Dynamics

In the paper [8] the author analyzes the influence of the mistuned τ_r^* on the flux dynamics and on the electrical torque produced by the machine. The analysis described in this chapter is generally based on this paper, with more attention paid on the influence of changing mutual inductance M with i_{ds}, and with the presumption that the rotor resistance R_r is constant and equal to the parameter R_r^* set in the controller. The important influence of R_r variation is analyzed separately (not in this article).

If the value of parameter τ_r^* set in the controller is different from the actual rotor time constant τ_r for $\Delta\tau_r = \tau_r^* - \tau_r$, as a consequence of the mistuned mutual inductance which has been changed for $\Delta M = M^* - M$, the rotor flux in the machine will differ from the one expected in the model. Besides, the q-component of the rotor flux (ψ_{qr}), which should be zero in case of correct d-q frame orientation, will become non-zero. It is possible to define the following flux components errors:

$\Delta\psi_{dr} = \psi_{dr}^* - \psi_{dr}$, where ψ_{dr}^* is the value in the controller

$\Delta\psi_{qr} = -\psi_{qr}$ or, in the matrix form:

$$\begin{bmatrix}\Delta\psi_{dr}\\ \Delta\psi_{qr}\end{bmatrix} = \begin{bmatrix}\psi_{dr}^*\\ 0\end{bmatrix} - \begin{bmatrix}\psi_{dr}\\ \psi_{qr}\end{bmatrix} \tag{1}$$

Since the d-component of the rotor flux is not zero, the model shown in Fig. 1. is not valid anymore. The equation system that now describes flux dynamics is:

$$\begin{bmatrix}\dot\psi_{dr}\\ \dot\psi_{qr}\end{bmatrix} = \begin{bmatrix}-\dfrac{1}{\tau_r} & \omega_s\\ -\omega_s & -\dfrac{1}{\tau_r}\end{bmatrix}\begin{bmatrix}\psi_{dr}\\ \psi_{qr}\end{bmatrix} + \frac{M}{\tau_r}\begin{bmatrix}i_{ds}\\ i_{qs}\end{bmatrix} \tag{2}$$

with dots over variables representing the first derivative of the variable.

The expression for the electrical torque is given by [1]:

$$T_e = \frac{3}{2}p_p\frac{M}{L_r}\left(\psi_{dr}\,i_{qs} - \psi_{qr}\,i_{ds}\right) \tag{3}$$

From the expressions (1) and (2), the flux error components are:

$$\begin{bmatrix}\dot\psi_{dr}\\ \dot\psi_{qr}\end{bmatrix} = \begin{bmatrix}-\dfrac{1}{\tau_r} & \omega_s\\ -\omega_s & -\dfrac{1}{\tau_r}\end{bmatrix}\begin{bmatrix}\psi_{dr}\\ \psi_{qr}\end{bmatrix} + \frac{M}{\tau_r}\begin{bmatrix}0\\ Mi_{qs}\dfrac{1}{\tau_r}\dfrac{\Delta M}{M}\end{bmatrix} \tag{4}$$

The solution of these differential equations in s-domain is:

$$\begin{aligned}\Delta\psi_{dr} &= M\,i_{qs}\frac{\Delta M}{M}\frac{\omega_s\tau_r}{s^2\tau_r + 2s\tau_r + (1+\omega_s^2\tau_r^2)}\\ \Delta\psi_{qr} &= M\,i_{qs}\frac{\Delta M}{M}\frac{1+s\tau_r}{s^2\tau_r + 2s\tau_r + (1+\omega_s^2\tau_r^2)}\end{aligned} \tag{5}$$

The error between the actual electrical torque and the electrical torque produced in the hypothetical model with the well oriented d-q frame is given by:

$$\Delta T_e = \frac{3}{2}p_p\frac{M}{L_r}\left(\Delta\psi_{dr}\,i_{qs} - \Delta\psi_{qr}\,i_{ds}\right) \tag{6}$$

with the flux components errors given in (5).

The expression (6) shows the influence of the mutual inductance error ΔM on the electrical torque produced by the machine. Several conclusions can be drawn from (6):

- The torque response to the i_{qs} change has an oscillatory character. The system is under-dumped (and stable), because the eigenvalues of the system given by:

$$s_{1,2} = -\frac{1}{\tau_r} \pm j\frac{1}{\tau_r}\frac{i_{qs}}{i_{ds}} = -\frac{1}{\tau_r} \pm j\frac{1}{\tau_r}a \tag{7}$$

- The damping factor is:

$$\xi = \frac{1}{\sqrt{1+\omega_s^2\tau_r^2}} = \frac{1}{\sqrt{1+\left(\dfrac{i_{qs}}{i_{ds}}\right)^2}} = \frac{1}{\sqrt{1+a^2}} \tag{8}$$

  with $\omega_s\tau_r = i_{qs}/i_{ds} = a$.
- The oscillations are more damped if the ratio i_{qs}/i_{ds} is smaller (eqn. 8). This means that the sensitivity of the field controlled drive generally increases with the load. With no load (i_q=0), errors in fluxes are zero, as well as in the torque. Higher power motors (having relatively lower magnetizing current) are more susceptible.
- Besides the undesirable oscillations, the steady state error exists as well. Both the flux components and the produced torque have the steady state error. Using the "load factor" a, defined in equation (8), the errors in the fluxes are:

$$\begin{aligned}\Delta\psi_{dr}^S &= \frac{\Delta M}{M}\frac{a^2}{1+a^2}\psi_{dr}\\ \Delta\psi_{qr}^S &= \frac{\Delta M}{M}\frac{a}{1+a^2}\psi_{dr}\end{aligned} \tag{9}$$

In equation (9), superscript ^{S} denotes steady state value, and ψ_{dr} is the d-component flux value that would exist if the mutual inductance M were constant and equal to the value set in the controller. In this case this flux component would be equal to the rotor flux amplitude.

The steady state rotor flux amplitude is:

$$\left|\psi_r^S\right| = \frac{\sqrt{1+a^2}}{\sqrt{1+\left(1+\dfrac{\Delta M}{M}\right)^2 a^2}}\left|\psi_{r0}\right| \tag{10}$$

with ψ_{ro} being the amplitude of the rotor flux that would be built in a well tuned controller. The electrical torque is:

$$T_e^S = \left(1+\frac{\Delta M}{M}\right)\frac{\left(1+a^2\right)}{1+\left(1+\dfrac{\Delta M}{M}\right)^2 a^2}T_{e0} \tag{11}$$

where T_{e0} is the electrical torque that would be achieved if the mutual inductance M were equal to the value set in the controller.

A simulation result of the torque response to the torque reference step from zero to the nominal value, with the flux producing current reference (I_{ds}^*) 1.2 and 1.5 times higher than nominal (requested rotor flux 20% and 50% above nominal) is shown in Fig. 4.

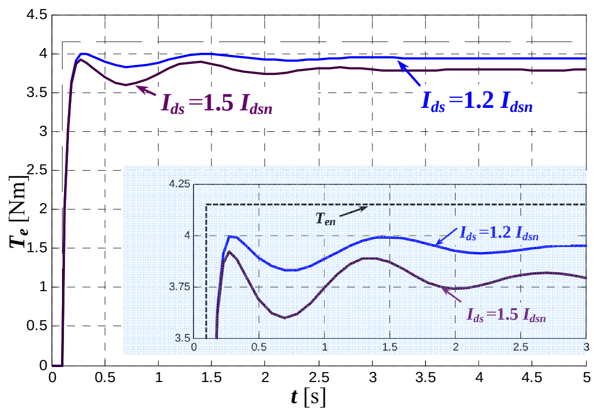

Fig. 4. Torque response to the reference step from zero to the nominal value with the requested rotor flux 20% and 50% above the nominal value.

The flux producing reference current (i_{ds}^*) has been kept constant at a value 20% and 50% higher than nominal. The reference torque step was made by a torque producing reference current (i_{qs}^*) step from zero to the value that provides the nominal torque at a given rotor flux. The command step takes place at t = 0,1s, when the flux has already been built. The simulation was made for experimental machine (chapter VI) using the expressions (3), (5) and (6).

The dashed line is a theoretical response for a well tuned controller. The embedded picture is an enlarged look at the step instant, shown to emphasize the delay and a shape of the response.

V. Mathematical Model Modification Saturated

The previous chapters show that the influence of saturation must be taken into account. It is possible to decrease the influence by improving the mathematical model if the magnetizing curve is known. There are numerous papers with proposed method for magnetizing curve identification at first start [11] and for the two parameter curve approximation [9] or tabulating.

This chapter is based on the results presented in [6] and proposes a slight simplification used in the experimental drive.

The rotor flux d-component ψ_{dr} can be split into two parts. The major part is a magnetizing flux ψ_m which depends on the magnetizing current i_m. The other, much smaller part is a leakage flux ($L_{\sigma r}\, i_{dr}$). The parameter $L_{\sigma r}$ can be treated as a constant (chapter II). With the assumption of the correct field orientation (ψ_{qr} = 0; ψ_{dr} = ψ_r), the d-component rotor voltage equilibrium (Fig. 5b) expression becomes:

$$0=\frac{d\psi_{dr}}{dt}+R_r i_{dr} \Rightarrow \frac{d\psi_{dr}}{dt}=-R_r i_{dr}=R_r\left(i_{ds}-i_m\right) \tag{12}$$

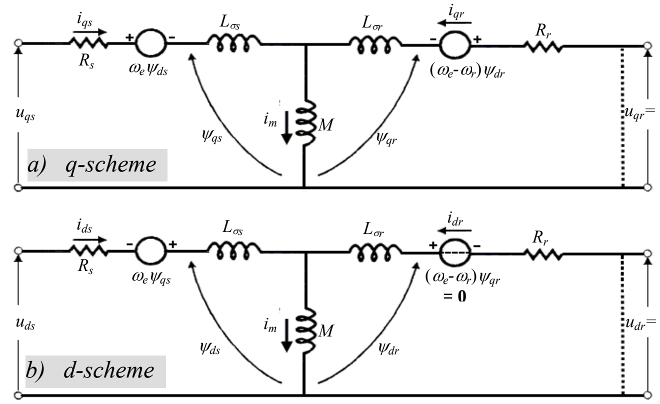

Fig. 5. Equivalent scheme of the machine in the d-q reference frame (a) q-component scheme, (b) d-component scheme. The schemes are independent of each other.

Magnetizing current i_m and magnetizing flux ψ_m are coupled by magnetizing curve (that mainly depends on motor construction and material) and i_m can be expressed as a function of ψ_m. This flux can be expressed as $\psi_m = \psi_{dr} - L_{\sigma r}\, i_{dr}$

$$i_m = f_1\left(\psi_m\right) = f_1\left(\psi_{dr}-L_{\sigma r}i_{dr}\right) \tag{13}$$

or, from (12):

$$\frac{d\psi_{dr}}{dt}=R_r i_{ds}-R_r f_1\left(\psi_{dr}-L_{\sigma r}i_{dr}\right) \tag{14}$$

From the expression (14), ψ_{dr} can be solved as a function of i_{ds}. Rotor current d-component i_{dr} is a function of i_{ds} and i_m.

Using (14) it is possible to modify the model given in Fig. 1. In the modified model (Fig. 6.), f_1 and f_2 are the tables in the controller.

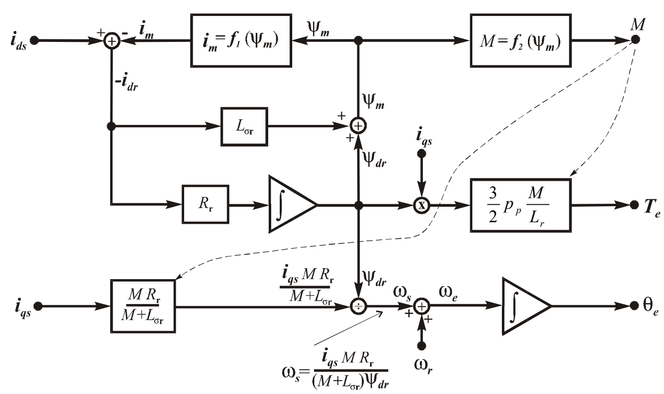

Fig. 6. Modified induction machine dynamics model in the d-q frame that respects the variation of M due to saturation.

Parameter M, used in some blocks, is now a dynamically adjusted variable read from table f_2. This table, denoted as $M = f_2(\psi_m)$ is derived from the known magnetizing curve $i_m = f_1(\psi_m)$ by calculating $M = \psi_m / i_m$ for certain ψ_m with i_m read from the first table. Input for both tables is ψ_m.

VI. Experimental Verification

To prove the conclusions, the experimental setup has been made using a standard induction machine coupled to a dynamic brake and fed from a three phase inverter controlled by a PC (Fig. 7.).

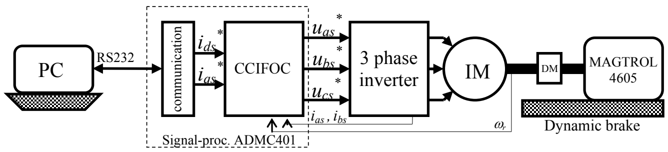

Fig. 7. Experimental setup. PC provides the reference values for the torque that motor should produce and for the rotor flux level. The dynamometer DM measures the produced torque.

The experimental setup is based on a standard four-pole, 0.75kW induction motor. The motor is fed from a three phase inverter with an indirect vector controller and current controllers of i_{ds} i i_{qs} in a synchronous *d-q* frame tied to the rotor flux vector. CCIFOC is implemented on a signal-processor ADMC401 and it is connected to a PC using RS232. PC controls the torque using i_{qs}^*, while the rotor flux is controlled by means of i_{ds}^*. For the particular motor, all parameter values, including the magnetizing curve are measured according to standard IEEE112. The tables f_1 and f_2 (chapter V) are derived from the experimentally obtained magnetizing curve.

Nominal torque of the motor is 4.15 Nm and is produced at the nominal slip speed ω_{snom} = 8 rad/s (f_{snom} = 1.27 Hz). Other parameters are as follows: stator resistance R_s = 3.35Ω; rotor resistance R_r = 1.99Ω; stator and rotor inductances L_s=L_r=170.7mH and the linear part mutual inductance M_0 = 163.7 mH. Rotor time constant is τ_r = 85.7ms.

The motor is mechanically coupled to the dynamic brake Magtrol 4605 [12] which is set to the maximum available torque (24 Nm). As the motor cannot produce 24 Nm, the brake practically blocks the shaft, making a standstill conditions (ω_r = 0) and in the experiment, only its dynamometer (Magtrol 4613 [12]) is used.

The experimental results for the torque reference step, identical to the step in the simulation (chapter 3), are shown in Fig. 8. and 9.

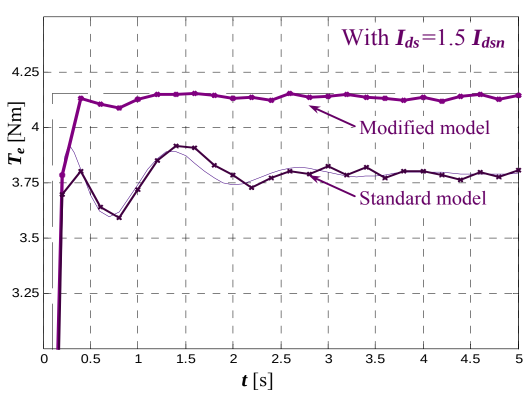

Fig. 8. The torque step response with the flux producing current 50% higher than nominal.

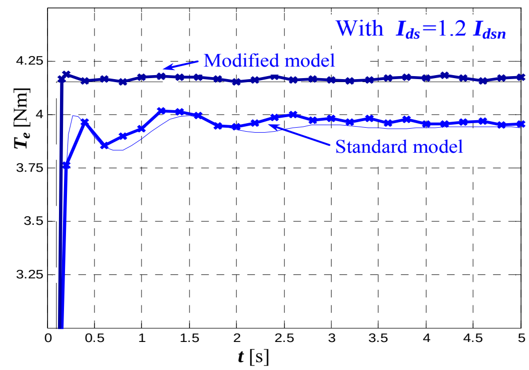

Fig. 9. The torque step response with the flux producing current 20% higher than nominal.

One curve is obtained using the standard model, while the other one is the result, using the modified model (shown in Fig. 6.). The thin line is repeated simulation result. The torque was sampled each 100ms. It should be stressed that the dynamometer input has an unavoidable second order low-pass filter with time constant of 10ms, so that measured values are slightly delayed from actual.

The modified scheme eliminates the influence of *M* variation only if the rotor resistance is tuned well, as it is in these experiments. Otherwise, the compensation is not full because the model uses i_{ds}^* and i_{qs}^* and not actual i_{ds} and i_{qs}.

VII. Conclusion

Induction motor drive with indirect vector control is sensitive to parameter variation. The critical parameter is rotor time constant, or mutual inductance and rotor resistance which dominantly define the rotor time constant. This paper concerns only the influence of the mutual inductance variation. The reason for this parameter variation is mainly (sometimes unavoidable) main flux path saturation.

Due to mutual inductance variation, produced electrical torque is different than expected, the response to command change is not instantaneous and have dumped oscillatory shape. It is possible to get partially solved the saturation problem by means of an improved model with a tabulated magnetizing curve.

The experiment showed close parallel with the simulation results.

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