

A New Real Differentiator with Low-Pass Filter in Control Loop

Čedomir Milosavljević, Branislava Peruničić, and Boban Veselić

Abstract—The paper offers a new real differentiator realized as a closed loop system with a low-pass filter (LPF) within the control loop. The cascade of the LPF, with a considerable time constant, and an integrator is treated as the controlled object. The controller is of derivative (D) type with a limiter and a high gain. The closed loop time constant of the obtained real differentiator is small. The limiter and the LPF significantly attenuate noises. Simulation comparison between the proposed differentiator and the known differentiators reveals certain advantages of the proposed solution under appropriate conditions.

Index Terms—Derivative estimator, Real differentiator, Sliding mode based differentiator.

I. INTRODUCTION

ESTIMATION of a real signal derivative is a well known and widely studied technical problem [1]-[9]. There are many approaches for a signal derivative estimation. The two main methods may be pointed out:

1. Model-based method;
2. Model-free method.

In the first method, the model of the process that generates the signal whose derivative is required is known. Typical model-free differentiators are (i) Euler's differentiator, (ii) real differentiator and (iii) sliding mode based differentiator [1]-[7].

As well known, Euler's differentiator is given by relation

$$\tilde{f}(t) = T^{-1}[f(kT) - f((k-1)T)]. \quad (1)$$

If $T \rightarrow 0$, $\tilde{f} \rightarrow \dot{f}$ by definition.

Unfortunately, if the input signal is noisy, the obtained differentiator's output will be contaminated as well. The amplitude of the input noise's derivative will be amplified if $T < 1$. Consequently Euler's differentiator is not recommended for a wide industrial application.

Consider an illustrative example [2] with an input signal

$$f(t) = \sin(\pi t), t \in [0, 2], T = 0.005s. \quad (2)$$

Fig. 1 shows the differentiation outputs of (a) noise-free input and (b) input contaminated with a uniform random noise having an amplitude of 0.004. It is obvious from the Fig. 1b that the Euler's differentiator cannot be used when the input is significantly contaminated by noises.

A real differentiator, defined by its transfer function as

$$\tilde{f}(s)/f(s) = s/(1 + \tau s), \quad (3)$$

where τ - is a time constant, is recommended for industrial applications. This differentiator has a low pass filter (LPF) $(1/(1 + \tau s))$. Its role is to attenuate high-frequency noises. Theoretically, in the noise-free case an accurate estimation can be obtained by choosing a small τ , Fig. 1a. However, in real systems with noisy inputs the constant τ must have a higher value which increases the phase lag. These contradictory requirements can be handled only by a adopting a compromise between the allowed level of the noise amplitude in the output signal and the phase lag.

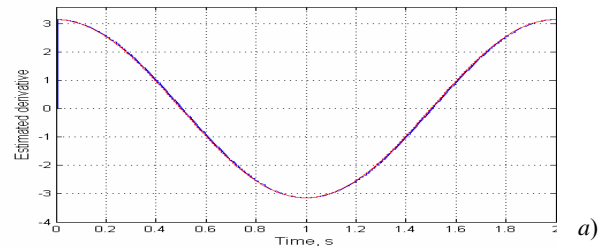


Fig. 1. a. Derivative of signal (2): a) noise-free case.

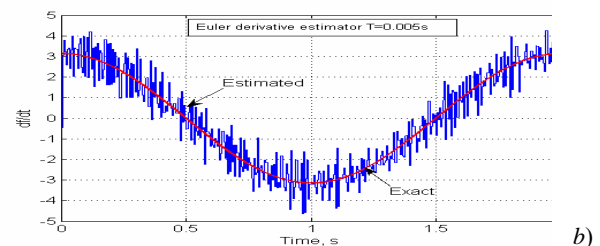


Fig. 1. b. Derivative of signal (2): b) Euler's differentiator noise case.

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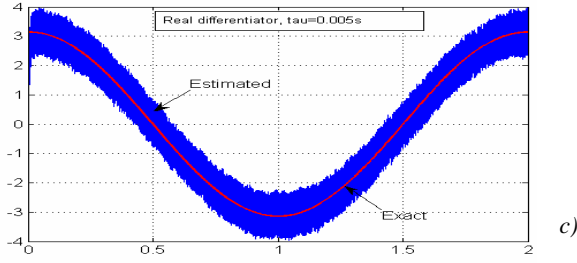


Fig. 1.c. Derivative of signal (2): a) noise-free case; b) Euler's differentiator noise case; c) Conventional real differentiator noise case. $T=\tau=0.005s$.

The real differentiator (3) is designed as an active control structure, Fig. 2. Derivative estimation of signal (2) using real differentiator with $\tau=0.005s$, in the noise contaminated case, is given in Fig. 1c.

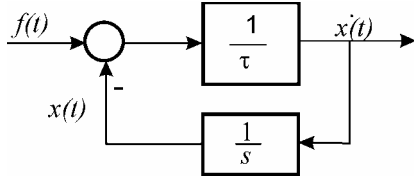


Fig. 2. Conventional real differentiator.

A particular problem arises when the input signal has low amplitude and/or a high noise/signal ratio. Then the estimated derivative cannot even be recognized.

As an example, let the input signal, $f(t) = 0.008\sin(\pi t)$, $t \in [0,2]$, be contaminated with the same noise as above. The output in this case is given in Fig. 3.

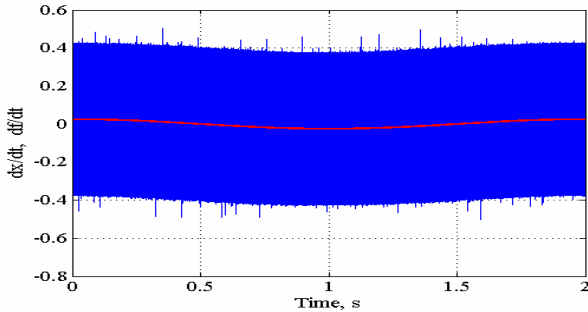


Fig. 3. Derivative estimation of signal $f(t) = 0.008\sin(\pi t)$, noisy-case, $\tau=0.005 s$.

It is clear that these derivative estimators cannot be used for wide range of applications. Pre-filtering or post-filtering significantly increases the phase lag so the estimated error will increase, also.

There are several solutions for the model-free derivative estimation. The focal part of this paper is sliding mode differentiators [1]-[7] explained in the Section 2. In the third section, a new real differentiator is proposed. It is realized as a conventional control loop. The controller is of the D-type and it has a high gain and saturation. An LPF between the controller and the output is introduced. Significant advantages of the proposed solution under certain conditions over differentiators [1] and [2] are demonstrated using simulations.

In the reminder of the paper the used noise parameters are identical to ones given above.

II. SLIDING MODE BASED DIFFERENTIATORS

Very high expectations have been set in application of sliding mode to obtain signal derivatives. The typical structure of these differentiators is given in Fig. 4. Their high-frequency switching nature requires introduction of an output LPF. Unfortunately, a LPF increases phase lag as well as estimation error.

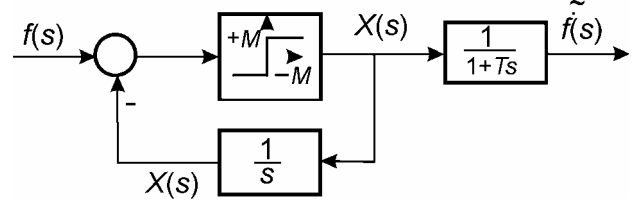


Fig. 4. Conventional sliding mode based differentiator.

Two sliding mode-based differentiators are of particular interest in this paper. The Levant's exact differentiation [1] is based on the second-order sliding mode approach. The differentiator proposed by Xu et al. [2], applies the closed loop filtering and uses the first-order sliding mode and the reaching law principle [10]. The next two subsections give essentials of these differentiators.

A. Levant's exact differentiator

Levant's differentiator, Fig. 5, is described by the following relations:

$$\dot{x}(t) = u(t), \tag{4a}$$

$$u = u_1 - \lambda |x(t) - f(t)|^{1/2} \text{sgn}(x(t) - f(t)), \tag{4b}$$

$$\dot{u}_1 = -\alpha \text{sgn}(x(t) - f(t)), \tag{4c}$$

where (4a) is differential equation of the "controlled object" – an integrator, and relations (4b) and (4c) describe second-order sliding mode controller. $f(t)$ is the input signal, whose derivative is estimated by control $u(t) = \dot{x}(t)$. $\alpha, \lambda > 0$ are gains whose values depend on the Lipchitz constant of the input signal $f(t)$. The main drawbacks of Levant's differentiator are the dependence of its parameters values of the input signal. To overcome this shortcoming an adaptive algorithm has been proposed in [4]. Another drawback is an overshoot at the beginning of the estimation.

As an example, Fig. 6 gives the output of the Levant's differentiator for the signal

$$f(t) = 5t + \sin(t) \rightarrow \dot{f}(t) = 5 + \cos(t); \lambda = 6; \alpha = 8, \tag{5}$$

where the parameters have been adopted from the original paper [1]. Runge-Kutta method with integration step of $10^{-4} s$ has been selected for the simulation.

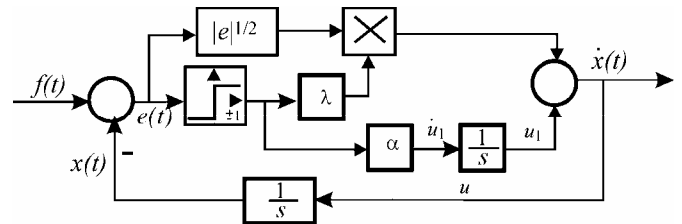


Fig. 5. Levant's exact differentiator.

Signal true differential and estimation obtained by Levant's differentiator for the noise-free case is given in Fig. 6a. The Fig 6b shows the estimation error. Its amplitude of $0.75 \cdot 10^{-3}$ may be taken as an excellent result. The noises in Fig. 6b are consequence of quantization resolution of $1,1921e-007$ they appear because a D/A converter is introduced in the simulation. However, Fig. 6c shows Levant's differentiator signal in the noise case. The noise in output signal is increased up to 0.4 (see Fig. 11a also). In such case, as was recommended in [1], it is necessary to use an additional LPF.

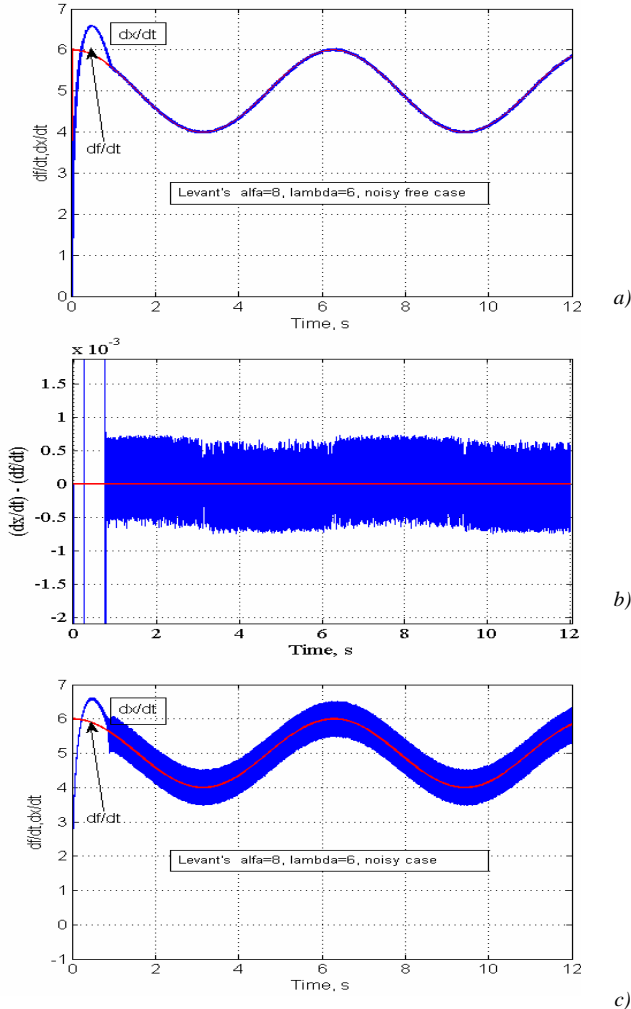


Fig. 6. Levant's differentiator: a) noise-free case; b) derivative estimation error for the noise-free case; c) contaminated case.

B. 2.2 Xu's et al. derivative estimator

This estimator is shown in Fig. 7.

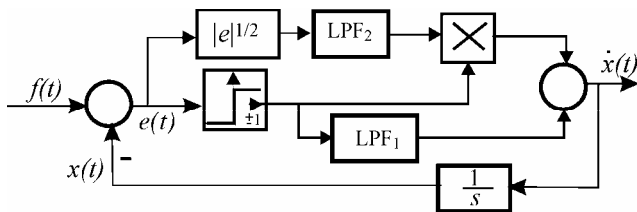


Fig. 7. Xu's et al derivative estimator [2].

Its main aims were to eliminate overshoot appearing in the Levant's differentiator and to minimize noise effects by

inserting a LPF in the closed-loop. Under nominal conditions, i.e. with no noise, it provides soft reaching of the sliding hyper-surface.

As an example in [2], differentiation of the signal was given.

$$f(t) = \sin(50\pi t); \dot{f}(t) = 50\pi \cos(50\pi t) \quad (6)$$

LPFs are described by the following transfer functions: LPF₁: $1000/(1+0.000055s)$; LPF₂: $1/(1+0.003s)$.

For Levant's differentiator and the signal (6) parameters α and λ are: 25000 and 180, respectively [2]. Simulation results of both estimators are given in Fig. 8a. Fig. 8b displays the expanded initial phase of estimation. The estimator of Xu et al. either does not have an overshoot at all, or a very small overshoot. The Levant's differentiator has a significant deviation from the exact derivative at the start of estimation.

Simulation for both differentiators with the same noisy input has similar outputs. Note very high signal amplitude, i.e. a very low noise/signal ratio. Also observe that the sign function in Xu's et al. estimator is replaced with saturation element.

For the higher noise/signal ratios, the estimator of Xu et al. gives a lower performance Fig. 9b, than the Levant's exact differentiator, Fig. 6c, for the same input signal (5). For a better estimation of the noisy signal derivative it is necessary to add one more LPF of Butterworth type [2] or to modify parameters in LPFs.

III. A NOVEL REAL DIFFERENTIATOR

In order to design derivative estimator that does not need parameters adjustment to the input signal $f(t)$ and to noise/signal ratio, a new simple structure is proposed in this paper, Fig. 10. This structure resembles the simple real differentiator structure from Fig. 2, but exhibits a more efficient noise filtration.

It is easy to deduce that, in the linear working regime, i.e. for small signals, the transfer function of the proposed differentiator is identical to the transfer function (3) with time constant $\tau = T/(1+K)$. For a higher gain K the system has a low phase distortion.

The noise in the input is significantly amplified by the block du/dt . Since the obtained signal passes through the limiter, the noise amplitude will be symmetrically saturated and thus averaged to zero. It is assumed that the useful signal's derivative stays in the linear zone of the limiter. The large time constant of the LPF averages noises, reducing significantly their impact on the output.

To verify the concept, the proposed differentiator and the Levant's differentiator have been tested with a variety of signals. The parameters of the proposed differentiator were: $K=10^5$, $T=10$ s, limiter saturation: ± 10 . Integration time in simulation is 10^{-4} s. MATLAB Simulink differentiator is used as du/dt element.

The differentiators have been firstly simulated with input signal defined by (5) in the presence of the above given noise.

For the Levant's differentiator parameters are chosen as suggested in [1]. Simulation results are in the Fig. 11.

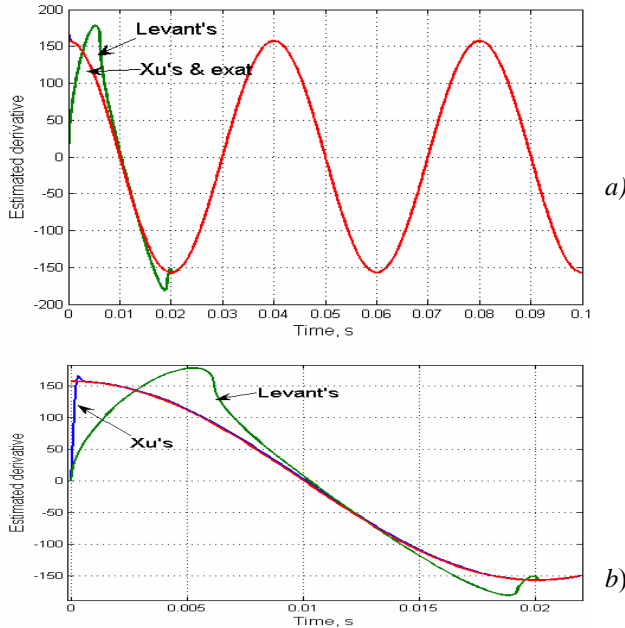


Fig. 8. Comparison of Levant's differentiator ($\alpha=25000, \lambda=180$) and Xu's et al. derivative estimator.

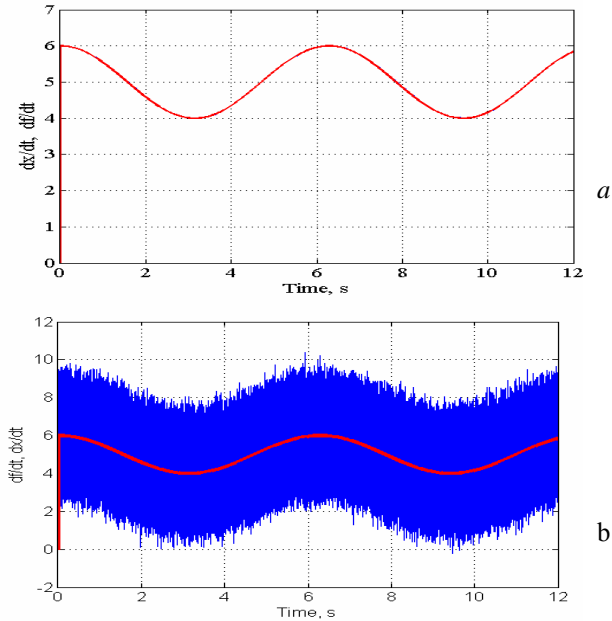


Fig.9. Xu's et al. derivative estimation for $f(t)=5t + \sin(t)$ a) noise-free case; b) noisy case.

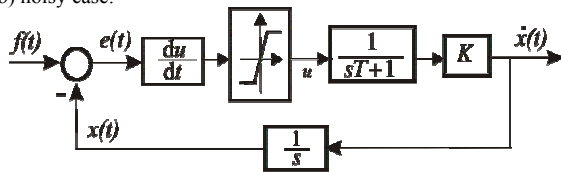


Fig. 10. Block-scheme of the proposed differentiator.

Fig. 11a displays the estimation error whose amplitude is near 0.4 for the Levant's differentiator. Response of the proposed differentiator depicted in Fig. 11b, indicates a shorter rise time and no overshoot in the response. Fig. 11c gives the estimation error of the proposed differentiator in the

presence of noise. Comparing the results from Fig. 11a and 6b it can be deduced that the Levant's differentiator has noise in the output even for a noise-free input. The amplitude of this noise is higher than the amplitude of the noises in the output of the proposed differentiator with a noisy input.

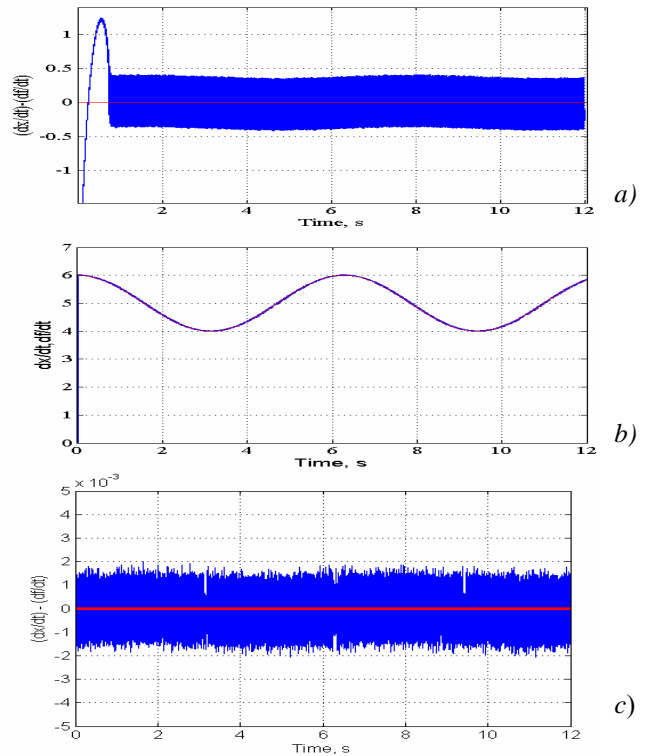


Fig. 11. Responses of differentiators for the signal (5), noise case: a) estimated error of Levant's differentiator; b) and c) response and estimated error of the proposed differentiator, respectively.

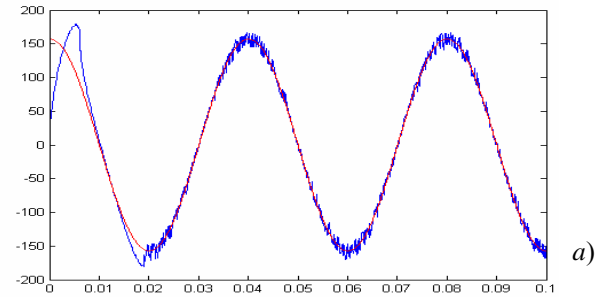


Fig. 12.a. Responses of differentiators for the signal $f(t) = \sin(50\pi)$ in the noisy case. a) Levant's differentiator.

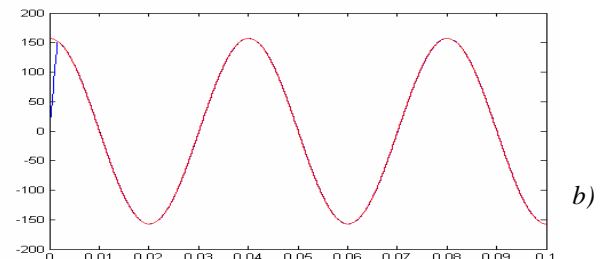


Fig. 12.b. Responses of differentiators for the signal $f(t) = \sin(50\pi)$ in the noisy case. b) the proposed differentiator.

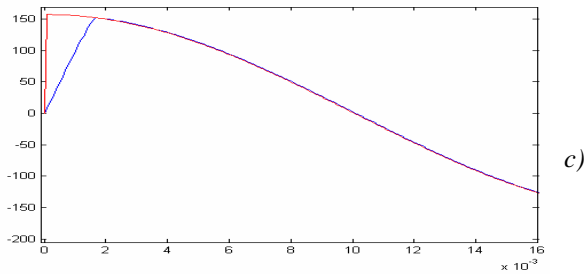


Fig. 12c. Responses of differentiators for the signal $f(t) = \sin(50\pi t)$ in the noisy case. c) Starting part of the response from Fig. 12.b (zoomed).

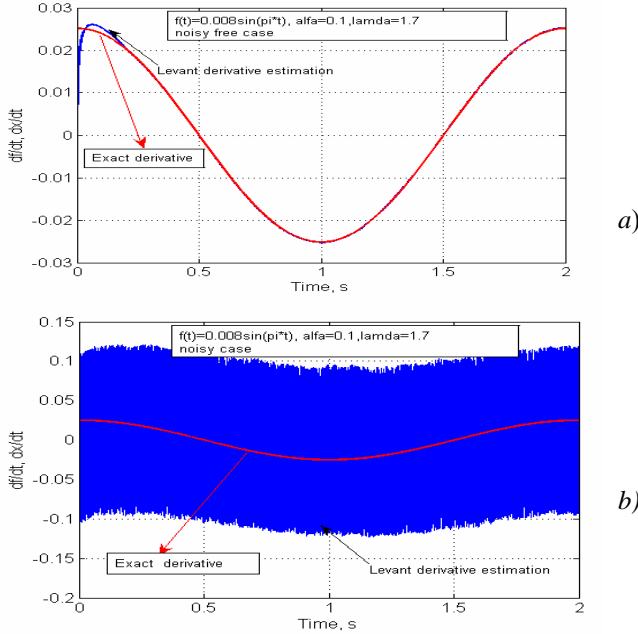


Fig. 13.a,b. Responses of differentiators for $f(t) = 0.008\sin(\pi t)$: a) and b) Levant's differentiator.

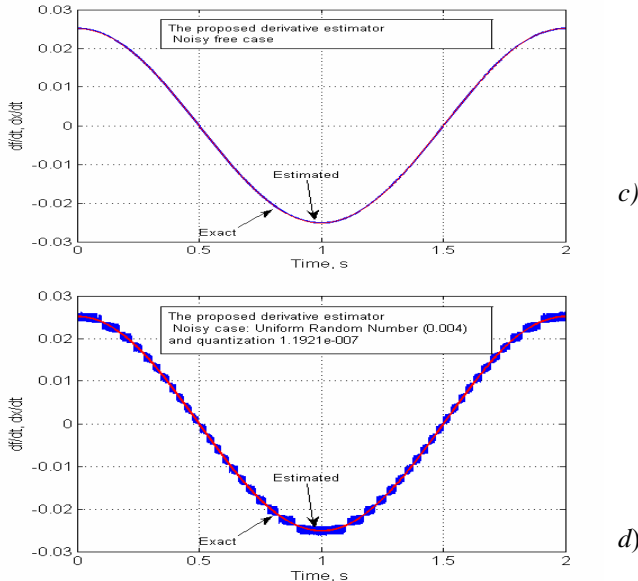


Fig. 13.c,d. Responses of differentiators for $f(t) = 0.008\sin(\pi t)$: c) and d) the proposed differentiator, noise-free and noise case respectively.

The second example is a differentiation of the signal (6) polluted with noise. Levant's differentiator parameters are chosen as in Fig. 8. The results are given in Fig. 12. Fig. 12a shows results of Levant's differentiator while the response of the proposed differentiator is given in Fig. 12b. Fig. 12c displays starting phase of the proposed derivative estimator. It is apparent that there are no overshoots.

In the next example a low amplitude signal $f(t) = 0.008\sin(\pi t)$ is used, with and without noise. The following parameters are chosen for the Levant's differentiator: $\alpha=0.1$; $\lambda=1.7$. Parameters of the proposed differentiator are not changed. Simulation results are given in Fig. 13. Fig. 13a and 13b display the response of the Levant's differentiator, for a noise-free and a noisy input respectively. Figure 13c and 13d show outputs of the proposed differentiator to the same signals.

Given simulation results indicate that the proposed differentiator has considerable better characteristics than Levant's differentiator except in one case. Namely, in Fig. 13d the resolution of D/A converter has the same consequence as a noise of step type in the output signal of the proposed differentiator. The analysis of simulation point out that the proposed differentiator requires a D/A converter resolution better than 24 bits. For the given study the resolution of 32 bits has been applied.

Operation of the Levant's and the proposed differentiator is given in Fig. 14 and Fig. 15 in the noisy case for the indicated signals, without any additional comments. Parameters of Levant's differentiator are specified in the figures, while parameters of the proposed differentiator are again unchanged.

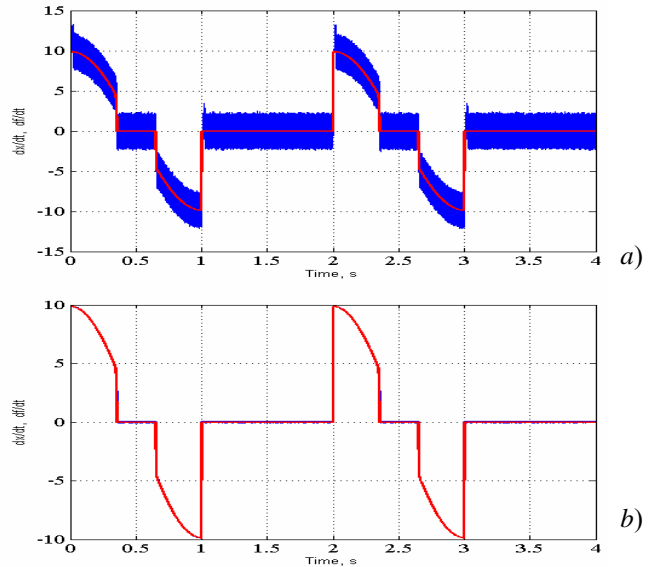


Fig. 14. Responses of differentiators for $f(t) = \pi \sin(\pi t)$ whose amplitude is saturated at ± 2.8 , in the noisy case: a) Levant's differentiator $\alpha = 500, \lambda = 35$; b) the proposed differentiator.

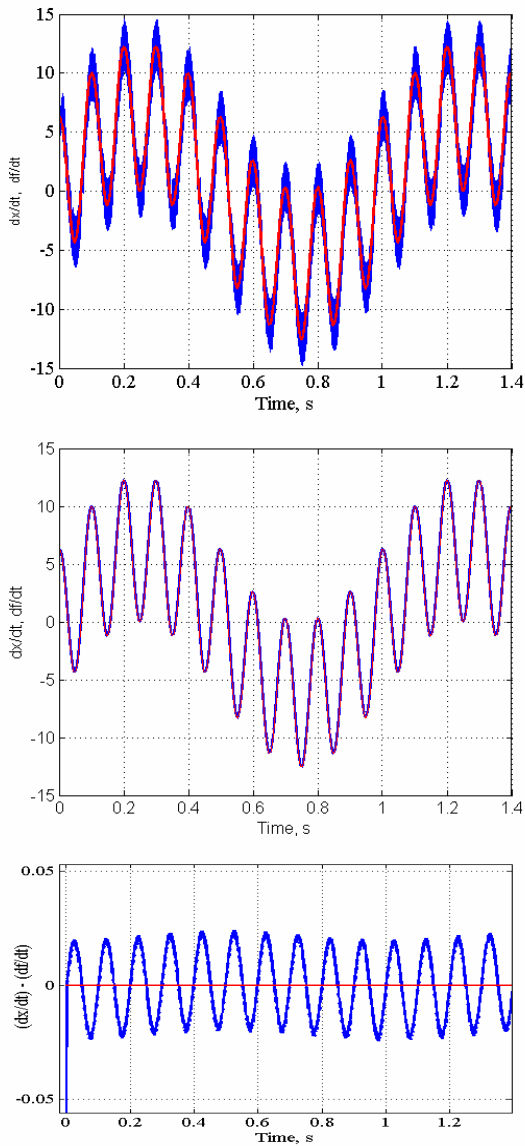


Fig. 15. Responses of differentiators, $f(t) = 1 + 0.1\sin(20t) - \cos(2\pi t)$, the noisy case: a) Levant's differentiator $\alpha = 500, \lambda = 35$ [4]; b) the proposed differentiator; c) estimation error of the proposed differentiator.

IV. CONCLUSION

A new control structure of real differentiator with LPF in the control loop is proposed in this paper. The proposed differentiator is compared with the well known Levant's exact differentiator. The obtained simulation with a variety of input signals show that the proposed differentiator shows significantly better results for to continuous-time input signals. For signals obtained from D/A converter the proposed differentiator needs a converter of very high resolution.

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