# Optimum Torque/Current Control of Dual-PMSM Single-VSI Drive

Andrea Del Pizzo, Diego Iannuzzi, and Ivan Spina

Abstract—The paper deals with isotropic PM-brushless drives in configuration "single-inverter, dual-motor" operating with unbalanced load-torques. An innovative control algorithm is presented. It is suitable to minimize the resultant armature current needed to obtain an assigned resultant motor torque, whatever is the load unbalance. Simplified analytical expressions are given in order to quickly evaluate optimized reference currents with good approximation. From these reference values, a predictive feeding algorithm evaluates the reference voltage space-vector for the inverter supplying the two motors in parallel. Current and torque oscillation, torque/current ratio, dynamic response and stability are the mainly observed quantities. Effectiveness of proposed control techniques is highlighted.

Index Terms-PMSM motor, dual motor, optimized control.

# I. INTRODUCTION

ECTOR and predictive control of ac drives are widely investigated and used in many application fields with reference to the classical configuration "single-inverter, singlemotor". A considerable interest has been also devoted to the control of drives composed by a single inverter feeding more motors in parallel. Main targets of these drives are reduced sizes and costs with respect to the single motor drives, either in industrial or in traction applications. Some scientific papers and practical applications can be found concerning singleinverter dual-motor drives which use induction motors, either with scalar or vector control [1, 2]. In these cases, for control purposes the parallel connected motors are assumed equivalent to one single motor. The majority of the papers refer to a dynamic machine model of the combined, parallel connected, dual induction motor system [3]. In addition, these papers illustrate torque-control methods based on the previous model, which enable mean and differential torque to be controlled during transient and steady-state operations.

The above described problems are not widely dealt with in the literature in case of PM-brushless motors supplied in parallel by a single inverter, as it can occur in those traction or industrial applications where PM motors are more and more requested. In these multi-motor drives either steady-state or transient operations could be more critical than in case of induction motors. This is due to the constancy of the rotor flux and to the absence of a rotor winding able to positively react in case of transient operations. In fact, considering two motors supplied in parallel by same frequency and voltage and with load unbalance, while in case of induction motors the rotor speeds get different values depending on the load-torque, in case of PM-brushless synchronous motors the rotor speeds are equal at steady-state, whatever is the unbalance values. As a consequence, when the load-torque of one motor suddenly varies, a risk of instability could occur if the angle between the armature voltage and e.m.f. vectors runs over  $\pi/2$ .

In the technical literature some authors have proposed a steady-state control of torque angle of only a motor a time, selecting that one with highest load-torque [4, 6, 7].

Other authors have suggested simple control configurations based on two main criteria: in a first case the two real motors are substituted by a single "equivalent" motor, by suitably handling measured current and speed values of each motor; in a second case the control and feeding algorithms are separately applied for the two motors and the resultant converter voltage is obtained by properly manipulating two voltage space-vectors separately evaluated. In this paper a new control technique is presented: on the basis of the measured speed and of the reference torque of both motors, the reference currents are evaluated in analytical way, imposing an optimizing condition to reduce the inverter size. A predictive feeding algorithm is used to evaluate the reference voltage space-vector for the inverter supplying the two motors in parallel.

# II. MATHEMATICAL MODEL

We refer to two isotropic PM-brushless motors supplied in parallel by a single inverter (Fig. 1).

We assume that parameters and rated values of motors "A" and "B" are equal, but each of them can be arbitrarily charged (unbalanced loads). Assuming sinusoidal the induced e.m.f. for both motors, their mathematical models in the respective rotor reference systems (superscript a or b) are expressed by:

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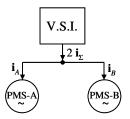


Fig. 1. Schematic representation of a single-inverter dual-motor configuration.

$$\begin{aligned}
\mathbf{v}_{A}^{a} &= R \cdot \mathbf{i}_{A}^{a} + L_{s} \frac{d}{dt} \mathbf{i}_{A}^{a} + jp \,\omega_{r,A} L_{s} \mathbf{i}_{A}^{a} + jp \,\omega_{r,A} \Phi_{r,A} \\
T_{A} - T_{L,A} &= J_{A} \frac{d}{dt} \omega_{r,A}; \ T_{A} &= \frac{3}{2} p \,\Phi_{r,A} \Im m \left\{ \mathbf{i}_{A}^{a} \right\} \\
\omega_{r,A} &= \frac{d}{dt} \vartheta_{A} \\
\end{aligned}$$

$$\begin{aligned}
\mathbf{v}_{B}^{b} &= R \cdot \mathbf{i}_{B}^{b} + L_{s} \frac{d}{dt} \mathbf{i}_{B}^{b} + jp \,\omega_{r,B} L_{s} \mathbf{i}_{B}^{b} + jp \,\omega_{r,B} \Phi_{r,B} \\
T_{B} - T_{L,B} &= J_{B} \frac{d}{dt} \omega_{r,B}; \ T_{B} &= \frac{3}{2} p \,\Phi_{r,B} \Im m \left\{ \mathbf{i}_{B}^{b} \right\} \\
\omega_{r,B} &= \frac{d}{dt} \vartheta_{B}
\end{aligned}$$

$$(1)$$

where  $\mathbf{i}_A$ ,  $\mathbf{i}_B$  are the current space-vectors of motors "A", "B", and the other symbols are explained in the list at the end of the paper. Denoting by  $2\psi$  the angular displacement between the axes of the rotor fluxes  $\Phi_A$  and  $\Phi_B$  (Fig. 2), the equivalent *d*axis is assumed in the mean position between  $\Phi_A$  and  $\Phi_B$ , while the flux magnitude produced by permanent magnets is the same ( $\Phi_{r,A} = \Phi_{r,B} = \Phi_r$ ), since we suppose the two motors equal.

In the new equivalent reference system (mean flux position), (1) become:

$$\begin{aligned} \mathbf{v}_{\Sigma} &= R \cdot \mathbf{i}_{\Sigma} + L_{s} \frac{d}{dt} \mathbf{i}_{\Sigma} + jp \,\omega_{r\Sigma} L_{s} \mathbf{i}_{\Sigma} + jp \,\omega_{r\Delta} L_{s} \mathbf{i}_{\Delta} + \\ &+ jp \,\omega_{r\Sigma} \,\Phi_{r} \cos \psi + p \,\omega_{r\Delta} \,\Phi_{r} \sin \psi \\ T_{\Sigma} - T_{L,\Sigma} &= J \frac{d}{dt} \,\omega_{r\Sigma} \quad \text{with:} T_{\Sigma} &= \frac{3}{2} \,p \,\Phi_{r} \left( i_{\Sigma q} \cos \psi + i_{\Delta d} \sin \psi \right) \\ 0 &= R \cdot \mathbf{i}_{\Delta} + L_{s} \frac{d}{dt} \mathbf{i}_{\Delta} + jp \,\omega_{r\Sigma} L_{s} \mathbf{i}_{\Delta} + jp \,\omega_{r\Delta} L_{s} \mathbf{i}_{\Sigma} + \\ &+ p \,\omega_{r\Sigma} \,\Phi_{r} \sin \psi + jp \,\omega_{r\Delta} \,\Phi_{r} \cos \psi \\ T_{\Delta} - T_{L,\Delta} &= J \frac{d}{dt} \,\omega_{r\Delta} \quad \text{with:} T_{\Delta} &= \frac{3}{2} \,p \,\Phi_{r} \left( i_{\Delta q} \cos \psi + i_{\Sigma d} \sin \psi \right) \end{aligned}$$

where subscripts " $\Sigma$ " and " $\Delta$ " refer respectively to "mean" and "differential" quantities, defined as (for a generic variable *G*):

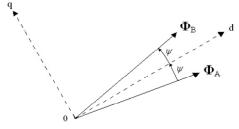


Fig. 2. Reference system.

$$G_{\Sigma} = \frac{G_A + G_B}{2} \quad ; \quad G_{\Delta} = \frac{G_A - G_B}{2}$$

In steady-state operations, denoting by  $\omega_r = \omega_{r,A} = \omega_{r,B}$  the rotor speed of both motors, the mathematical model is given by:

$$\begin{cases} \mathbf{V}_{\Sigma} = R \cdot \mathbf{I}_{\Sigma} + jp \,\omega_r L_s \mathbf{I}_{\Sigma} + jp \,\omega_r \,\Phi_r \cos \psi \\ 0 = R \cdot \mathbf{I}_{\Delta} + jp \,\omega_r L_s \mathbf{I}_{\Delta} + p \,\omega_r \,\Phi_r \sin \psi \\ T_{\Sigma} = \frac{3}{2} p \,\Phi_r \left( I_{\Sigma q} \cos \psi + I_{\Delta d} \sin \psi \right) \\ T_{\Delta} = \frac{3}{2} p \,\Phi_r \left( I_{\Delta q} \cos \psi + I_{\Sigma d} \sin \psi \right). \end{cases}$$
(2)

In a control problem,  $\omega$ ,  $T_{\Sigma}$  and  $T_{\Delta}$  are known quantities; then, the system (2) corresponds to a set of six real equations with seven real unknown quantities ( $\mathbf{V}_{\Sigma}, \mathbf{I}_{\Sigma}, \mathbf{I}_{\Delta}, \boldsymbol{\psi}$ ). Due to this degree of freedom, one 'auxiliary condition' is needed in order to solve the system (2). This condition could represent the *control algorithm* for the considered dual drive.

# **III. CURRENT CONTROL ALGORITHM**

A noticeable quantity is the torque to current ratio:

$$\rho = \frac{T_A + T_B}{\left|\mathbf{I}_A + \mathbf{I}_B\right|} = \frac{T_{\Sigma}}{\left|\mathbf{I}_{\Sigma}\right|} = \frac{T_{\Sigma}}{I_{\Sigma}}.$$
(3)

On the basis of (2)  $\rho$  can be expressed as:  $A \sin \psi \cos \psi$ 

$$\rho = \frac{1}{\sqrt{\left(B\cos\psi + C\sin\psi\cos^2\psi\right)^2 + \left(D\sin\psi + E\sin^3\psi\right)^2}}$$
$$\to \rho = \rho\left(T_A, \frac{T_B}{T_A}, \omega_r, \psi\right)$$

where:

$$A = T_{\Sigma} 3p \Phi_r Z^2; \quad B = 2T_{\Delta} Z^2; \quad C = -3p^3 \omega_r^2 \Phi_r^2 L$$
$$D = 2T_{\Sigma} Z^2 \quad ; \quad E = 3p^2 \omega_r \Phi_r^2 R; \quad Z = \sqrt{R^2 + (p\omega_r L)^2}.$$

The qualitative curve  $\rho(\psi)$  is plotted in Fig. 3a for assigned values of  $\omega_r$ ,  $T_A$  and of load-unbalance  $r=T_B/T_A$ . This behavior suggests to state as control algorithm the condition:

$$\max\left\{\frac{T_{\Sigma}}{I_{\Sigma}}\right\} \quad \Rightarrow \quad \frac{\partial}{\partial\psi}\left(\frac{T_{\Sigma}}{I_{\Sigma}}\right) = \frac{\partial\rho}{\partial\psi} = 0 \tag{4}$$

which is also suitable to minimize the size of the feeding inverter.

In case of balanced shaft loads, (4) corresponds to separate maximization of both torque/current ratios  $\{T_A/|\mathbf{I}_A|\}$  and  $\{T_B/|\mathbf{I}_B|\}$  for the two motors.

From (2) it is easy to verify that the d-q components of the "mean" current  $\mathbf{I}_{\Sigma} = I_{\Sigma d} + j I_{\Sigma q}$  are:

$$\begin{cases} I_{\Sigma d} = \frac{2T_{\Delta}}{3p\Phi_r} \frac{1}{\sin\psi} - \frac{(p\omega_r)^2 \Phi_r L}{R^2 + (p\omega_r L)^2} \cos\psi \\ I_{\Sigma q} = \frac{2T_{\Sigma}}{3p\Phi_r} \frac{1}{\cos\psi} + \frac{p\omega_r \Phi_r R}{R^2 + (p\omega_r L)^2} \frac{\sin^2\psi}{\cos\psi}. \end{cases}$$
(5)

By substituting (5) in (4) we deduce that the derivative (4) becomes a transcendent function of  $\psi$ , not solvable in analytical way. However, we can demonstrate that the function  $\rho = T_{\Sigma}/I_{\Sigma}$  is positive in the interval  $\psi \in (0, \pi/2)$  and  $\rho=0$  either for  $\psi=0$  or  $\psi=\pi/2$ . Consequently,  $\rho$  has a maximum  $\rho_{optim}$  in the interval  $(0, \pi/2)$  in correspondence of a  $\psi_{optim}$  value (see Fig. 3a. Diagrams b, c and d of Fig. 3 show the shifting of the point ( $\psi_{optim}$ ,  $\rho_{optim}$ ) in correspondence of different values of either  $r = T_B/T_A$ , or  $\omega$  or  $T_A$  respectively.

With reference to a pair of equal motors, whose rated parameters are in Table I (see section VI), Fig. 4 show the behaviour  $\rho_{optim}$  against *r* (i.e. for different unbalanced conditions) in correspondence of different load torques  $T_A$  and for two different steady-state rotor speeds  $\omega_r$  (continuous lines).

In correspondence of relatively low values of load unbalance, the angle  $\psi_{optim}$  is enough small to assume:  $\sin \psi \cong \psi$ ;  $\cos \psi \cong 1$ . (6)

Introducing this approximation, (4) becomes a 1<sup>st</sup> degree equation in  $\psi$  and a very simple analytical expression of  $\psi_{optim}$  can be found:

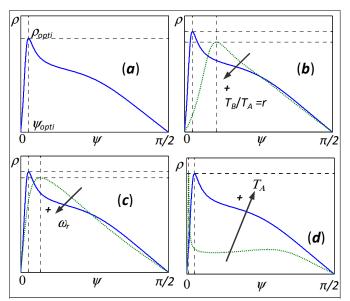


Fig. 3. Behavior of  $\rho$  as a function of  $\psi$  (a) and for different values of r,  $\omega_r$ ,  $T_A$  (b), (c), and (d).

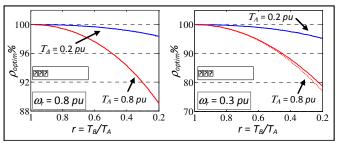


Fig. 4. Behavior of  $\rho_{optim}$  % as a function of the load-unbalance *r*, for two different speed values and for low and high load.

$$\psi_{optim}^{(a)} = \frac{2T_{\Delta} \left[ R^2 + \left( p \omega_r L \right)^2 \right]}{3p^3 \omega_r^2 \Phi_r^2 L} ; \text{with } T_{\Delta} = T_A \frac{1-r}{2}$$
(7)

where  $\psi_{optim}^{(a)}$  is the approximated value of  $\psi$  which maximizes the  $\rho = T_{\Sigma}/I_{\Sigma}$  ratio. It is independent on  $T_{\Sigma}$  and can be analytically determined. The correspondent curves of approximated  $\rho_{optim}$  are plotted against *r* in Fig. 4 with dotted lines. As we can see, they are generally very close to the not simplified curves and differ from them only for low values of *r* ( $\equiv$  high load unbalance).

The validity of the approximation (6) is also confirmed by the compared behaviours of  $\psi_{optim}$  and  $\psi_{optim}^{(a)}$  in Fig. 5 which refers to the same cases analysed in Fig. 4.

From Figs. 4 and 5, we also deduce that as higher load unbalance is, as lower the maximum value of  $\rho_{optim}$  and higher the shift angle  $\psi_{optim}$ .

In correspondence of the optimized condition (3), in Fig. 6 magnitude  $I_{\Sigma}$  of the mean current is drawn against unbalance r, for different speed and load conditions.

These values are compared to the approximated ones, obtained using (5)-(7), and to the ones obtained using a different control technique, named "one motor control", better explained in section V [4]. While the differences between

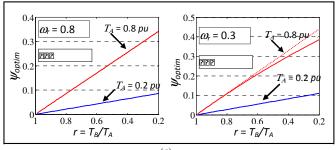


Fig. 5. Behavior of  $\Psi_{optim}$  and  $\Psi_{optim}^{(a)}$  as a function of the load-unbalance *r*, for two different speed and load-torque values.

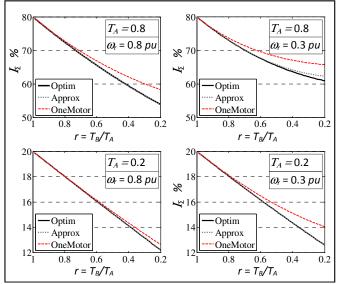


Fig. 6. Magnitude of the mean current  $I_{\Sigma}$  for different control techniques in some speed and torque operating condition.

exact and approximated  $I_{\Sigma}$  curves are very small, the current  $I_{\Sigma}$ assumes considerably higher values in the case of the "one motor control" technique, with negative consequences on the converter size.

## IV. PREDICTIVE FEEDING ALGORITHM

A predictive feeding algorithm can be useful to reduce current and torque distortion with respect to the use of hysteresis or PI current controllers [5].

We consider the discrete stator model of isotropic PM motors with all the electric quantities expressed in the mean *d*,*q* reference frame:

$$L_{s} \frac{d}{dt} \mathbf{i} + \dot{Z}_{s} \mathbf{i} = \mathbf{v}_{n} - \boldsymbol{\varepsilon}_{n}; \text{ for } t \in (t_{n}, t_{n+1})$$
(8)

where:  $\dot{Z}_s = R + j p \omega_r L_s$ ;  $\mathbf{\epsilon}_n = j p \omega_{r,n} \Phi_r \cos \psi_n$ . In (8)  $\mathbf{v}_n$  and  $\mathbf{e}_n$  are evaluated at a generic sampling instant  $t_n$ . During the interval  $(t_n, t_{n+1})$ , thanks to some suitable assumptions in the model (8), we can evaluate the reference voltage  $\mathbf{v}_{n+1}^* = v_{d,n+1}^* + j v_{q,n+1}^*$  to be applied at the instant  $t_{n+1}$ , in order to obtain – at the instant  $t_{n+2}$  – armature current  $\mathbf{i}_{n+2}$ equal to the reference current at  $t_n$  (i.e.,  $\mathbf{i}_{n+2} = \mathbf{i}_n^*$ ). We have:

$$\mathbf{v}_{n+1}^{*} = \mathbf{\varepsilon}_{n} + \dot{Z}_{s} \frac{\dot{\mathbf{i}}_{\Sigma,n}^{*} - \dot{\alpha}_{t} \left[ \dot{\alpha}_{t} \left( \mathbf{i}_{\Sigma,n} - \frac{\mathbf{v}_{n} - \mathbf{\varepsilon}_{n}}{\dot{Z}_{s}} \right) + \frac{\mathbf{v}_{n} - \mathbf{\varepsilon}_{n}}{\dot{Z}_{s}} \right]}{\frac{\dot{Z}_{s} \Delta t}{\frac{1 - \dot{\alpha}_{t}}{\frac{1 - \dot{\alpha}_{t}}{\frac$$

The feeding algorithm (9) needs only the knowledge of the reference current  $\mathbf{i}_{\Sigma,n}^* = i_{\Sigma d,n}^* + j \, i_{\Sigma q,n}^*$  and of the motor state in  $t_n$  [i.e.:  $\psi_n, i_{\Sigma d,n}, i_{\Sigma q,n}, \omega_{r,n} = (\omega_{r,A,n} + \omega_{r,B,n})/2$ ].

## V. CONTROL DIAGRAM

The used speed control circuit is described in Fig. 7.

The actual speed of each motor is separately detected and compared with the imposed  $\omega_r^*$  speed value. From the two reference torques  $T_A^*$  and  $T_B^*$ , the block  $C_1$  evaluates the mean and differential reference torques  $T_{\Sigma}^{*}$ ,  $T_{\Delta}^{*}$  and, in cascade, C<sub>2</sub>

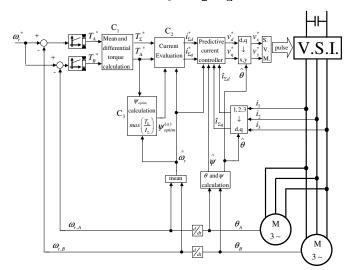


Fig. 7. Proposed control circuit.

evaluates the reference currents  $i_{\Sigma d}^*$ ,  $i_{\Sigma q}^*$  by means of (5) and using  $\psi_{optim}^{(a)}$  calculated by block C<sub>3</sub> corresponding to (7). An encoder on each rotor shaft separately detects actual values of position and speed. The estimated d,q components  $\hat{i}_{\Sigma d}, \hat{i}_{\Sigma q}$  of the actual mean currents are derived from measurement of the resultant inverter currents.

Reference and actual currents, together with mean actual speed  $\hat{\omega}_r$ , are used by the "predictive current controller" to evaluate the reference voltages  $v_d^*$ ,  $v_q^*$ , which are transformed in stator coordinates x,y using the mean angular position  $\hat{\theta} = (\theta_A + \theta_B)/2$ . Comparing the proposed control technique with the ones in literature [6] we can observe that in Fig. 7 the reference currents  $\hat{i}_{\Sigma d}, \hat{i}_{\Sigma q}$  are evaluated by imposing an optimizing analytical criterion while in [6] they are obtained simply adding the values separately obtained for the two motors. Another advantage of the proposed control is that only the feedback of the resultant stator currents is used (instead of a double current loop).

In the next section, the numerical results of the "proposed control technique" are compared to the one obtained using the simpler control diagram of Fig. 8 ("one-motor control"), as proposed in the literature [4].

#### VI. NUMERICAL ANALYSIS

A numerical analysis is carried out with reference to two three-phase PM brushless motors having equal rated values, as summarized in Table I.

The average switching frequency of the IGBT-SVM voltage source inverter is about 2 kHz. The load torque characteristic of both motors is assumed linear in function of the angular speed. Reference speed is set to  $\omega_r^* = 0.8 \omega_{r,R}$  for the case in Fig. 9.

Actual speeds, torques and currents of both motors A and B are plotted in Fig. 9, together with the mean currents  $i_{\Sigma d}$ ,  $i_{\Sigma a}$ , the voltage amplitude and the shifting angle between the two rotor polar axes.

The letters inside the figure have the following meaning: A) actual speeds of both motors; B) electromagnetic and load

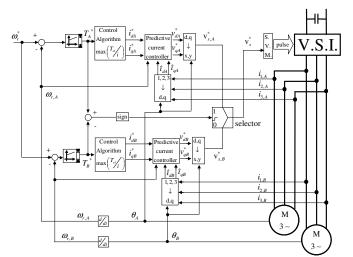


Fig. 8. One-motor control circuit.

 TABLE I

 MAIN DATA OF BOTH PM MOTORS

Rated power $P_R$	74 kW	Rated speed $\omega_{r,R}$	$\frac{33.5}{1}$ rad s <sup>-</sup>
Pole-pair p	8	Rotor inertia $J_r$	$0.9 \ kgm^2$
Rated voltage $V_R$	570 V	Armature inductance	5.7 mH
Rated current $I_R$	128 A	Armature resistance R	0.27 Ω

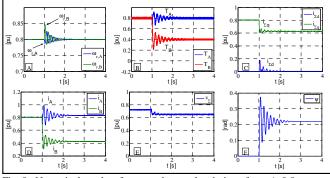


Fig. 9. Numerical results of proposed control technique for  $\omega_r \approx 0.8$  pu

torque of both motors; C) *d*-axis and *q*-axis components of the mean current; D) current magnitude of both motors; E) magnitude of reference voltage space-vector; F) electrical angle  $\psi$  between the two rotor frames.

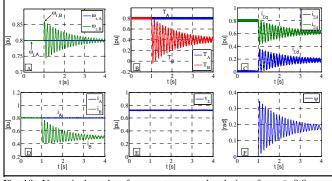
The load conditions are equal for both motors until the instant t=1 s, and correspond to load-torque values  $T_{A,L}=T_{B,L}=0.8$  pu of the rated torque. At the instant t=1 s, a step variation from 0.8 to 0.4 pu (50%) of the load torque is introduced for motor B, in order to test the capability of the system to get a steady-state condition together with acceptable values of amplitude both of torque and current oscillations. From the diagrams we can deduce that the decrease of the load torque  $T_{B,L}$  produces transient variations in all the electromechanical quantities of both motors A and B.

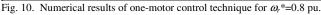
Referring to Fig. 9, when steady-state is reached, the speeds of the two motors assume again the initial value 0.8  $\omega_{r,R}$ ; the electromagnetic torques of both motors follow the respective load values; the *d*-component  $i_{\Sigma d}$  of the mean current is practically equal to zero, while the *q*-component  $i_{\Sigma q}$  decreases of about 25%. The angular positions of the two rotors assume different values and the shift-angle between them remains constant.

The diagrams of Fig. 10 represent the same quantities of Fig. 9, in equal operating conditions, but with reference to the control circuit of Fig. 8 ("one-motor control").

The following Figs. 11 and 12 show analogous quantities of Figs. 9 and 10 with reference to low speed operating condition.

Comparing the results of Figs. 9 and 11 to the ones of Figs. 10 and 12, it is easy to deduce that the "proposed control technique" (circuit in Fig. 7) gives rise to better performance both in steady-state and dynamic operations, with respect to the "one-motor control" circuit in Fig. 8. In fact,  $i_{\Sigma d}$  in Figs. 10c and 12c assumes steady-state values considerably higher than in the correspondent ones in Figs. 9c and 11c. Moreover, it is evident the greater time needed by the





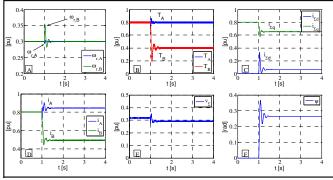


Fig. 11. Numerical results of proposed control technique for  $\omega_r^*=0.3$  pu.

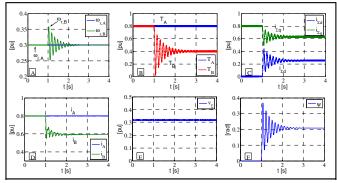


Fig. 12. Numerical results of one-motor control technique for  $\omega_r \approx 0.3$  pu.

"one-motor control" together with a greater magnitude of torque, speed and current oscillations.

## VII. SHORT CONCLUSIVE REMARKS

With reference to a drive composed by a single inverter feeding two isotropic PM brushless motors in parallel, a new control technique is proposed in order to manage load unbalances. It is based on a control algorithm and a feeding algorithm in cascade. The feeding algorithm uses the predictive voltage evaluation already presented in [6] that is able to obtain low values of current distortion and torque pulsation. The control algorithm is based on an auxiliary condition which optimizes the set of two motors, aiming to maximize the ratio (resultant torque)/(resultant current).

The main features of the proposed control are: a reduced number of current transducers, steady-state operations with the minimum input current for every resultant load-torque in case of load unbalance (reduction of inverter size), reduced overshoot and acceptable dynamics in transient operations, good stability also in presence of heavy variations of the load torque on only one motor.

#### APPENDIX

- $\mathbf{I}_A, \mathbf{I}_B$  steady-state current of motor A, B
- $\mathbf{I}_{\Sigma}, \mathbf{I}_{\Lambda}$  steady-state mean and differential current
- *L* armature inductance
- *p* pole-pair
- *R* armature phase resistance

 $T_{A,L}, T_{B,L}$  load torque of motor A, B

- $\theta_A; \theta_B$  angular position of rotor A, B
- $\Phi_r$  air-gap flux of rotor magnets
- $\omega_r$  rotor angular speed
- $\omega_{r,R}$  rated value of rotor angular speed
- $\psi_{optim}$  optimized shift angle between rotor polar axes

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