The Dependence of Resistivity on Temperature for Thin Superconductors

Constantin Blaj, Dumitru Toader, and Marian Greconici

***Abstract*—Measurements made on superconducting very thin layers are analysed by modelization using 2D FEM. The electric field distribution is established and can be seen the differences of this distribution during the superconductive transition. The calculation of the temperature variation of the resistivity is made based on measured temperature variation of the resistance, taking into account also the electro kinetic field distribution.**

***Index Terms*—Superconductivity, very thin layers, resistivity measurements.**

I. Introduction

MEASUREMENTS made in liquid nitrogen on YBaCuO samples show the transition of high critical temperature superconductors to the superconductive stage. Very thin samples (60...130 microns thickness), tapes of 5-6 mm width and 50-60 mm length are measured using V-A measurement method. To the samples are soldered electric conductors trough thin drops of silver, in order to assure good electric contacts for the electric measurements. When measuring the voltage and the current, by their ratio, it can be calculated the resistance of a compound resistor. By modelization and using FEM calculation it is analysed how precise the curve representing the variation of the resistance with the temperature fits to the curve showing the variation of the resistivity of the thin superconductor with temperature.

The transition from normal resistor to superconductor it is produced in a 2-3 K temperature interval, as can be seen in the experimental curve presented in Fig. 1. In between the two temperatures, at the beginning at the end of the transition to the superconductive stage (temperatures marked as T_a and T_b) exist the inflexion point of the curve and the temperature corresponding to this point, T_c, can be assumed to be the critical temperature of the superconductive layer.

In Fig. 2 is presented the theoretical curve for variation of the resistivity of the high critical temperature superconductor (curve 1) and the variation of the resistivity of silver (2) in the region of 100…120 K. Between the temperatures T_a and T_b the silver's resistivity has a slight modification, and shall be considered constant in the following calculations [1].

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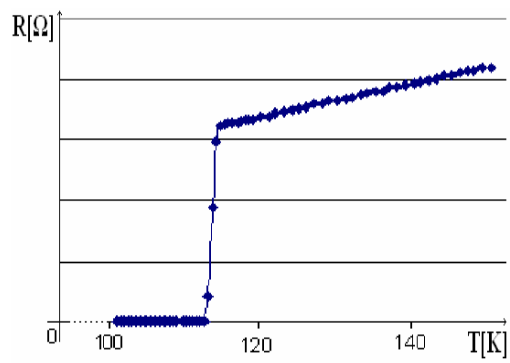

Fig. 1. Resistance of the sample vs. temperature.

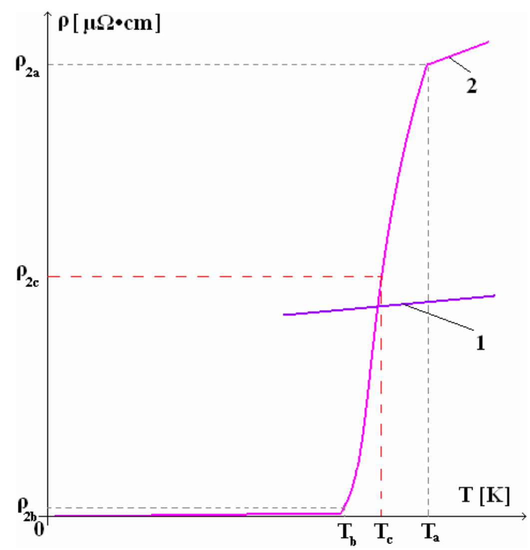

Fig. 2. Resistivity for YbaCuO (2) and silver (1).

II. The Resistance of a Compound Resistor

The thin layer of high critical temperature superconductor, together with the small drops of high conductivity silver, represents a compound resistor [4].

The two silver contacts are very important in size because of the small thickness of the superconductive layer [7].

In Fig. 3 is presented the situations of the measurement with only two contact points. It is not convenient to use the same contacts for injecting the current as well as to measure the voltage difference between them [8]. Some of the inconvenient of using this method shall be presented also in this paper.

The four points measurement method can be used for

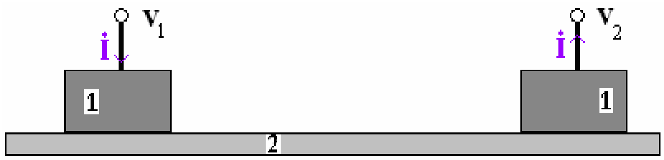

Fig. 3. The two contacts measurement method.

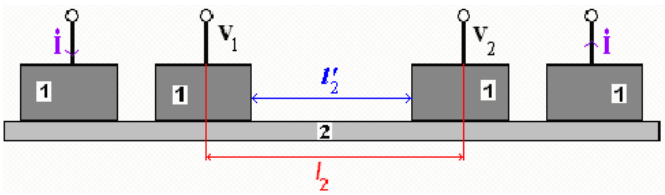

Fig. 4. The four contacts measurement method.

calculating a resistance as the ratio between the voltage measured between two electrodes and the current injected trough other two electrodes

$$R = \frac{V_1 - V_2}{I}. \qquad (1)$$

This resistance, in certain conditions, can be accepted as being the resistance of the thin tape of length l_2.

During the transition to the superconductive stage, by decreasing the temperature of the sample immersed in liquid nitrogen, the resistivity of the YBaCuO layer decreases slightly [3] until the T_a temperature. At this temperature the resistivity of silver used for the electric contacts is several times smaller than the resistivity of the tape.

From the temperature T_a descending to T_b the resistivity of the tape decreases abruptly in a few seconds, even if the temperature interval is only of about 2…3 K. The resistivity of the silver can be considered constant in this very narrow temperature interval. At a certain moment, corresponding to a temperature between T_a and T_b, the two resistivities, of the silver and of the tape, are equal one to the other. As the temperature decreases towards T_b the resistivity of the tape becomes smaller and smaller. At T_b the resistivity of silver is several times greater than the resistivity of the tape. After a few seconds the resistivity of the tape is reaching finally the zero value (superconductive stage of the tape) and remains constant even if the decreasing of the temperature goes on.

When the current flows through a compound resistor at the surface separating the two media there are involved some continuity conditions for the electric field.

The distribution of the electric field in this compound resistor modifies significantly during the transition from temperature T_a (when $\rho_1 < \rho_2$) to T_b (when $\rho_1 > \rho_2$).

In Fig. 5, it is shown the continuity of the current density from medium 2 to medium 1, when $\sigma_1 > \sigma_2$ ($\rho_1 < \rho_2$). In Fig. 6, it is shown the situation when the resistivity of medium 2 has decreased and became smaller than the resistivity of medium 1 (silver contact).

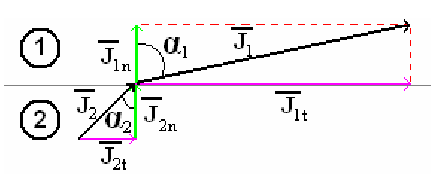

Fig. 5.

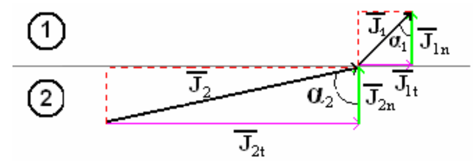

Fig. 6.

When the thin layer, medium 2, reaches the superconductive stage the current under the voltage contact has only horizontal component and the current does not enter inside the silver voltage contact, medium 1. This situation is presented in Fig. 7a. Because the current has a finite value inside the layer 2, even if the conductivity $\sigma_2 = \infty$ for temperatures $T < T_b$, this implies that the electric field $E_2 = 0$.

In Fig. 7b is presented the situation under the current injection silver contact. The surface that separates the two media is equipotential surface.

III. The Numeric Modelization of the Compound Resistor

The two measurement methods shown in Figs. 3 and 4 were modelised in 2D FEM, using QField Terra Analysis Student version [5]. For the principle of the method, the 250 nodes are enough to give an image to the students on how measurement results for calculating the electric resistance can be combined with numeric modelisation of the compound resistor. Using both methods the final result (the temperature dependence of the resistivity of a studied material) can be obtained. Also the accuracy of the 4 contacts method is proved by the modelisation also.

As shown in Fig. 2 during the short temperature interval, ($T_a \rightarrow T_b$) the resistivity of the superconducting layer modifies dramatically while the silver's resistivity remains practically the same (we consider it constant in the followings). So, from a situation when the ratio between the resistivities of the two materials is smaller than 1, at the temperature T_a, $k(T_a) = \rho_1(T_a)/\rho_2(T_a) < 1$, by reducing the temperature we pass through a point where the ratio is $k(T) = \rho_1(T)/\rho_2(T) = 1$, and we arrive at temperature T_b where $k(T_b) = \rho_1(T_b)/\rho_2(T_b) >> 1$. Some field distribution representation show the important difference imposed by the variation of the resistivity. Due to symmetry only a half of the model was represented. The representation is made for the situation of the measurement

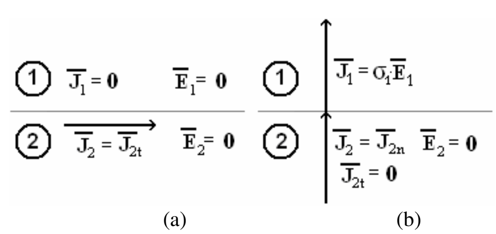

Fig. 7.

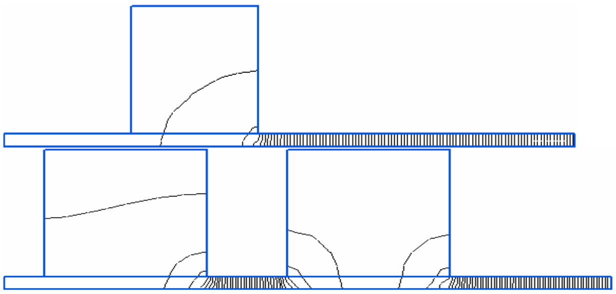

Fig. 8. Representation of the equipotential lines for k=0.2.

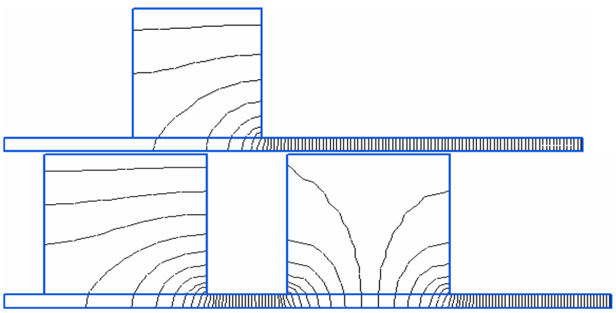

Fig. 9. Representation of the equipotential lines for k=1.

with two contacts as well as with four contacts.

IV. THE CALCULATION OF THE RESISTIVITY CURVE $P_2(T)$

If we assume that the field between the voltage contacts is uniform in each point of the thin layer on the length l_2, the curve of the resistivity is obtained by proportionality from the measured curve of the resistance $R_{ms} = f(T)$.

This is valid for greater values of $k = \rho_1(T)/\rho_2(T)$. For k = 5, situation presented in Fig. 10, we can see that this condition is nearly fulfilled.

If we take as reference the resistance of the sample measured at T_b, one of the last points on the descending resistance curve before reaching the superconductive stage, for this situation $k \approx 50$ and the resistivity can be calculated with (2).

$$\rho_2(T_b) = \frac{R_{ms}(T_b)\cdot S_2}{l_2}\,. \tag{2}$$

For the analysed sample this value, calculated based on the electric measurements and geometric dimensions of the sample, is $\rho_2(112.75)$ = 0.102 nΩ·m.

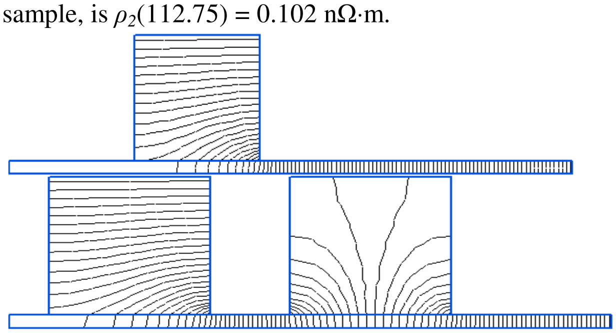

Fig. 10. Representation of the equipotential lines for k=5.

At the same temperature the resistivity of silver is $\rho_{Ag}(112.75) = \rho_2(112.75) \approx 5$ nΩ·m [2].

The relative value of the resistance measured at another temperature, T in the interval (T_b, T_a), is the resistance. R^*_{ms}, and their values are calculated in the third row of Table I.

Starting from the situation corresponding to k = 50, and using the exact dimensions of the sample (including contacts) several numeric modelizations were made. One of the results of modelization is presented in Fig. 11. Based on the symmetry, only half of the compound resistor was modelized.

The resistance is calculated using the field distribution obtained with QField for each modelization (k from 50 to 0.21) and results are in Table II.

$$R_{md} = \frac{2\cdot\int_m^n \overline{E}\cdot\overline{dl}}{\int_S \overline{J}\cdot\overline{dS}} = \frac{2V_m}{i} \tag{3}$$

V_n = 0 and the value of V_m is read from the program.

The current is calculated, using again the program's facilities, by integrating on surface S.

From the measurements (Table I), we have a dependence on temperature of the relative resistance of the sample.

From modelization (Table II) we have a dependence of the resistance of the compound resistor on the resistivity of the thin YBCO layer. The reference value in the two tabs corresponds to the same situation, the calculated one and the modelized one. For this situation which corresponds to a temperature T_a = 112.75 K, the resistivity of YBCO is $\rho_2(112.75)$ = 0.102 nΩ·m.

The measured resistance (the relative value) of the compound resistor corresponds to a certain ratio between the resistivity of the silver and the resistivity of the thin YBCO layer. Because was assumed that for the rapid transition to the superconducting stage the resistivity of the silver can be accepted as constant, this means that the relative resistance of the compound resistor is function of the resistivity of the YBCO layer.

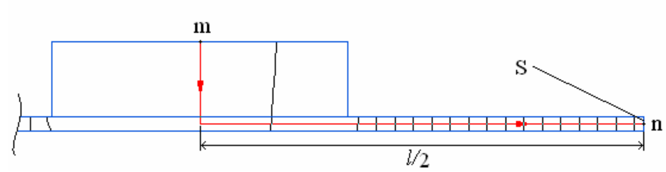

Fig. 11. Modelization of the measured sample.

TABLE I

T [K]	112.75	113.14	113.78	114.24	114.77
R_{ms} [nΩ]	1.074	21.829	99.401	156.17	169.55
R^*_{ms} [-]	1.0	20.325	92.552	145.41	157.87

TABLE II

k	50	5	1	0.5	0.33	0.25	0.21
ρ_2[nΩ·m]	0.1	1	5	10	15	20	24
R^*_{md}[-]	1	8.82	38.2	76.2	107.8	142.4	170

$$R^* = f_1(T) \text{ and } \rho_{2md} = f_2(R^*) \tag{4}$$

Using (5), based on measured R_{ms}, can be obtained the dependence on temperature of the resistivity of YBCO assuming uniform electric field in the layer:

$$\rho_{2ms}(T) = \frac{R_{ms}(T) \cdot S_2}{l_2}. \tag{5}$$

Using relations (4), we obtain the dependence on temperature of the resistivity of the YBCO based on modelization and on the dependence on temperature of the relative resistance of the compound resistor, $\rho_{2md}(T)$. Both of them are presented in Table III.

In graphic representation the variation of the resistivity is presented in Fig. 12. It can be seen that exist a difference between the two calculated resistivities.

V. CONCLUSIONS

By the numeric modelization, using QField 2D FEM, the field distribution shows very clearly that the four contact measurement method is much more accurate than the two contact method.

Using the approximation that the electric field inside the thin YBCO layer is uniform, the value of the resistivity results by simple calculation (5), assuming that it is proportional to the resistance calculated from measurements. For very low values of the resistivity of the thin film this is correct, but for medium values and especially for high values of the resistivity this is not correct.

If the resistivity of the silver is much smaller than the resistivity of the YBCO thin layer (at high, "normal", temperatures) the distance l'_2 can be used instead of l_2 and the results would be more accurate. This can be seen from the equipotential field lines represented in Fig. 8.

TABLE III

T	ρ_{md}	T	ρ_{ms}
[K]	[nΩ·m]	[K]	[nΩ·m]
112.750	0.1	112.75	0.102
112.906	1.0	113.14	2.079
113.298	5.0	113.78	9.467
113.611	10.0	114.24	14.874
113.916	15.0	114.77	16.148
114.213	20.0	115.36	16.298
114.700	22.0	117.08	16.458
117.982	23.0	118.04	16.698
120.800	24.0	120.14	16.955

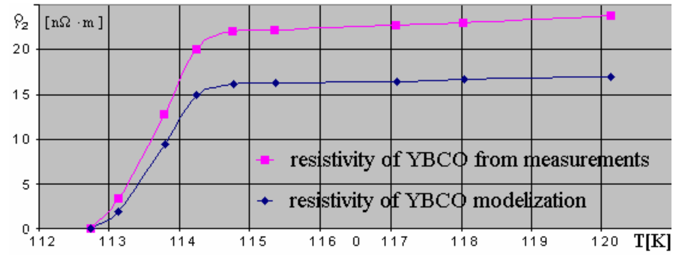

Fig. 12.

But using l'_2 instead of l_2 gives unsatisfying results when the two resisitivities are of the same range and surely shall give incorrect results for the values of the resistivity when the YBCO approaches to the superconducting stage.

By technical point of view the differences between the two curves of the resistivity are not very significant, but if a very accurate numeric modelization is made the result can be a more accurate curve $\rho(T)$.

Some of the authors [6] when presenting results on thin superconducting film give the dimensions of the cross section of the film (width and thickness) and a result as resistance per unit length, R^* [nΩ/m] as function of the temperature, $R^*(T)$.

The critical temperature T_c shows to be in fact a little bit smaller but is not modified in a significant manner by the correction proposed by this paper.

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