Upgrade of Conventional Positional Systems into High-Precision Tracking Systems Using Sliding Mode Controlled Active Digital Compensators

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***Abstract*—The paper offers a possibility of upgrading conventional PD controlled positional systems into high-precision tracking systems using active compensators. For improving of tracking as well as disturbance rejection capabilities of these systems, two digital active compensators are used. The first one is feedforward improvement of tracking, whereas the second one represents feedback compensation of disturbances. The introduced compensators contain active sliding mode controlled subsystems. The proposed solution does not require any additional sensors. The proposed control extension is described as well as digital sliding mode controller design procedures. Also, simulation results in case of dc motor servo-system are presented.**

***Index Terms*— Servo-systems, Active compensators, Sliding mode control, Digital controllers.**

I. Introduction

POSITIONING and tracking are the two basic control tasks that can be met in motion control. In positioning the input or referent signal is step function. It is required to provide as fast and accurate response as possible, preferably without overshoot, whereas the transient trajectory is not specified. In tracking it is necessary to enforce the system output to continuously and accurately follow the referent signal, which may represent very complex trajectory. Modern production technologies impose on control systems more rigorous demands. One of them is flexibility, meaning that the same positional servo-system equally successfully execute the both afore mentioned control tasks, under action of parameter variations and external disturbances.

Most of positional systems in mechatronics, robotics and various industrial applications are realized by using conventional PD controllers. Such systems can ideally track only constant signals, but already under action of constant external disturbance the positioning error occurs. In the applications where an accurate tracking of complex trajectories is required under action of disturbances, these systems give unsatisfactory results. Then some other control technique must be applied that provides simultaneously both accurate tracking and great robustness. One approach may be use of the two degree of freedom controllers, which allow the problems of tracking and disturbance rejection to be treated separately. Moreover, it is possible to independently tune the responses with respect to the referent signal and to the disturbances, [1]-[3]. Further improvement is suggested in [4,5] by multirate sampling.

An appropriate solution to the described control task is implementation of variable structure control systems (VSCS) [6], whose theoretical invariance to disturbances in ideal sliding mode [7] is reduced to excellent robustness in practical realizations. That is the reason why VSCS found their largest application exactly in this field. As a state space technique, VSCS need information of all state coordinates for ideal tracking of arbitrary referent signals. This practically means the knowledge of the tracking error signal and its successive derivatives, and therefore the knowledge of referent signal derivatives. Accordingly, ideal tracking is possible only for the analytically known or known in advance references. Since this is not the case in servo-systems, tracking accuracy depends on a number of available derivatives of the tracking error signal [8]. Second order sliding mode control is suggested in [10] for the servo-system synthesis, where sliding mode based differentiator is used for evaluation of the error signal derivative [11,12]. Differentiators are practically useful only for the first and second order derivatives of the signal, whereas high order derivatives are completely inapplicable due to severe noise contamination.

In order to further improve system accuracy additional disturbance compensation is often carried out. Extraordinary improvements were achieved in various servo-systems by so called active disturbance estimator (ADE) [9,13], which contains a sliding mode controlled active subsystem. Also, there is a possibility of introduction of supplemental integral action into VSCS that additionally increases system

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accuracy [14].

This paper proposes a way to upgrade the conventional positioning systems into a high-accuracy robust tracking systems by using active compensators (ACs). Since a conventional system needs to be improved in tracking capability as well as in disturbance rejection, two digital ACs are introduced. The first AC represents feedforward improvement of tracking. The second AC is actually the ADE [9,13] that compensates system disturbances and is located in a local feedback loop. These digitally implemented ACs involve an active control substructure based on discrete-time sliding mode control (DSMC). In the paper the proposed control extension is described in details, DSM controller design procedure is explained and simulation tests on DC motor are presented.

II. Ideal Tracking System

The well-known control structure with feedforward and disturbance compensations is shown in Fig.1 in digital realization. Under certain conditions this structure can ensure the output signal to ideally track the reference.

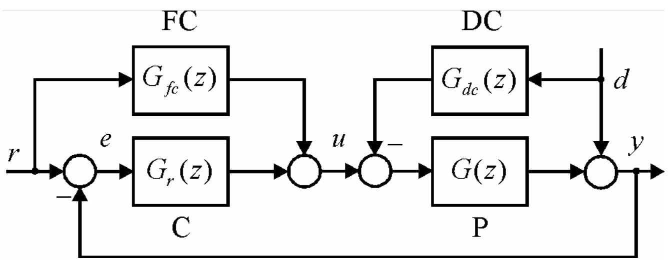


Fig. 1. Block scheme of ideal tracking system: P-plant; C-main controller; DC-disturbance compensator; FC-feedforward compensator.

According to the structure in Fig. 1, the error signal may be easily expressed in complex domain with respect to reference and disturbance :

$$E(z)=\frac{[1-G(z)G_{fc}(z)]R(z)+[G(z)G_{dc}(z)-1]D(z)}{1+G_r(z)G(z)}. \quad (1)$$

Ideal tracking occurs when the tracking error is annulated $(e(k)=0)$, which is the case when it holds

$$G_{fc}(z)=G^{-1}(z)\wedge G_{dc}(z)=G^{-1}(z). \quad (2)$$

Hence, in order to achieve ideal tracking it is necessary that the transfer functions of the feedforward and disturbance compensators represent plant inverse dynamics. This requirement inevitably raises the following questions:

- how to obtain the information about disturbance if it is not available for direct measurement?

- how to overcome plant parameters uncertainty and variations as well as unmodeled dynamics.?

- how to realize plant inverse dynamics, since it is not a causal system?

The answers to these questions are offered in the following consideration.

A. Disturbances Estimation and Compensation

Information about external disturbances is practically impossible to obtain by direct measurement. Therefore it is necessary to estimate the disturbance for its compensation. One possible structure for disturbance estimation is presented in Fig. 2a. In this digital realization extraction of the equivalent disturbance $q(k)$ is done using the nominal plant model $G_n(z)$. Mismatch between the nominal model and real plant inevitably exists due to parameter variations and unmodeled dynamics. Hence, plant dynamics may be described as

$$G(z)=G_n(z)(1+\delta G(z)), \quad (3)$$

where perturbations are limited by the multiplicative bound of uncertainty $\left|\delta G(e^{j\omega T})\right|\le\gamma(\omega),\ \omega\in[0,\pi/T]$. The extracted equivalent disturbance is obtained in the form

$$q(k)=d(k)+G_n(z)\delta G(z)u_k(k), \quad (4)$$

indicating that the equivalent disturbance carries information about external disturbance, which can always be mapped onto plant output, and parameter perturbations and unmodeled dynamics, i.e. internal disturbances.

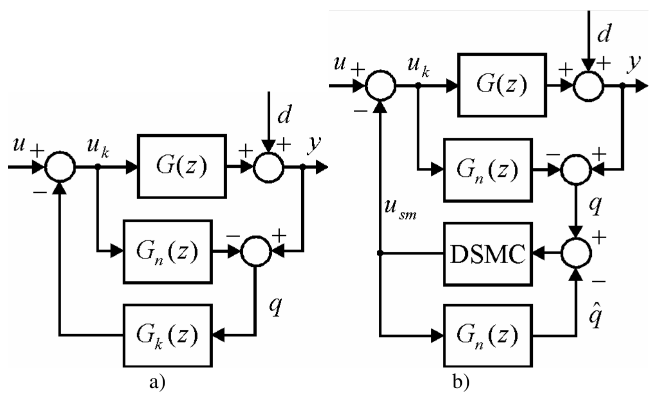


Fig. 2. a) Disturbance estimator; b) ADE based on DSMC.

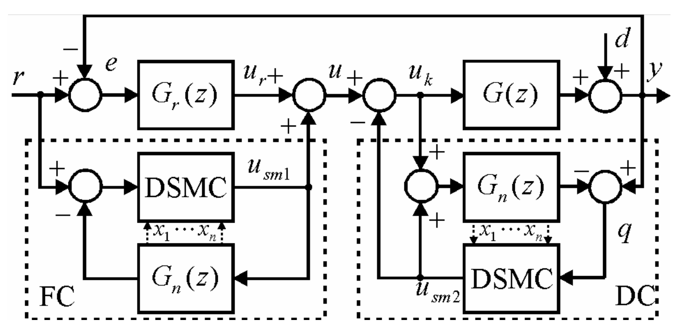


Fig. 3. The proposed control extension of the conventional system.

According to Fig. 2a, plant output as a function of control and disturbance may be expressed as

$$Y(z)=\frac{G_n(z)(1+\delta G(z))}{1+G_k(z)G_n(z)\delta G(z)}U(z)+\frac{1-G_k(z)G_n(z)}{1+G_k(z)G_n(z)\delta G(z)}D(z) \quad (5)$$

If the compensation filter $G_k(z)$ represents nominal plant inverse dynamics, i.e. $G_k(z) = G_n^{-1}(z)$, output becomes $U(z) = G_n(z)U(z)$, which shows that all disturbances are completely eliminated and that the nominal plant behavior is ensured. Unfortunately, such filter is non-causal and cannot be realized.

Solution is proposed in [9] through the concept of ADE, Fig. 2b, where passive filter is replaced by an actively controlled subsystem. If DSM controller within ADE provides $\hat{q}(k) = q(k)$, i.e. ensures an ideal DSM regime, then controller output may be described as $U_{sm}(k) = G_n^{-1}(z)Q(z)$, showing that this subsystem acts as nominal plant inverse dynamics. Thus, complete disturbance rejection is achieved nominal plant behavior is secured. This way transforms the disturbance compensation problem into tracking problem of the referent signal $q(k)$. In the tracking subsystem of ADE, DSM controller governs nominal model, not the real plant, so there are not any uncertainties and all state coordinates are available. Generally, due to the not known in advance referent signal $q(k)$ it is possible to establish only quasi-sliding regime [15], resulting in nonideal disturbance rejection. However, since DSMC systems provide high-accuracy tracking, an excellent compensation may be expected, i.e. near nominal behavior.

B. *Active Compensators*

The notion from ADE may also be used in realization of inverse dynamics that is required by the feedforward compensator FC in Fig. 1, which should improve reference tracking capacity of the system. Theoretically designed structure in Fig. 1 may be practically realized as shown in Fig. 3.

Disturbance compensator DC is actually an observer variant of ADE, which is formed by optimization of the structure in Fig. 2b. If DSM controller within FC establishes discrete sliding mode, then it holds $U_{sm1}(z) = G_n^{-1}(z)R(z)$, which shows that FC acts as nominal model inverse dynamics. Since DC ensures plant nominal behavior, the resulting system exhibits ideal tracking of arbitrary referent signal. This control extension is suitable for the upgrade of already existing systems, whose main conventional controller G_r, usually PD type, gives modest performance in tracking of complex signals in the presence of disturbances. Retuning of the main controller is not necessary, since the proposed control structure requires the main controller to be designed for the nominal plant, which is already the case in the practice.

Although it is known that SMC systems need measurement of all state coordinates, such system expansion does not require any additional sensors. Only input/output measurements are required, since ACs contain nominal models, which provide necessary state information for the DSM controllers. Stability of the overall system is secured by the occurrence and existence of the sliding regimes in the DSMC subsystems.

III. DSMC Design

Both DSM controllers within KP and PK govern the nominal model and may be identical. The priority is to ensure as accurate tracking as possible in order to gain the precise nominal model inverse dynamics. Good results were obtained using DSMC algorithm [14], which guaranteed ideal tracking of parabolic signals. This control algorithm is based on the algorithm [16] enriched with the introduction of additional integral action with respect to the switching variable. Integration is activated only in the predefined vicinity of the sliding surface. Emergence of chattering, extremely undesirable phenomenon in SMC systems, is eliminated by imposing a linear control zone near the sliding surface [16]. Since the exposed control algorithm has been thoroughly elaborated in [14] and [13], it will be briefly described hereafter.

Let the nominal plant model be in the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u_{sm} \Rightarrow \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u_{sm},\ \hat{q} = x_1 . \quad (6)$$

It is a second order model representing the mechanical subsystem dynamics of an electromechanical positional system. The dynamics of the electrical part, i.e. electric drive, is neglected since it is much faster than the mechanical counterpart. Tracking error may be calculated as $e(t) = q(t) - \hat{q}(t) = q(t) - x_1(t)$. The model (6) may be transformed into the tracking error space

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} - \mathbf{b}u_{sm} + \mathbf{p},\ \mathbf{e} = \begin{bmatrix} e_1 = e \\ e_2 = \dot{e} \end{bmatrix}, \mathbf{p} = \begin{bmatrix} 0 \\ a\dot{q} + \ddot{q} \end{bmatrix}. \quad (7)$$

Unlike the previous model, a disturbance vector **p** occurs in this model as a consequence of variability of the input signal q. The first component $a\dot{q}$ scan be easily compensated since forming of e_2 already needs knowledge of $\dot{q}$, which may be obtained using a differentiator. However, $\ddot{q}$ cannot be reliably obtained by twofold differentiation due to drastic amplification of noises. Hence, vector $\mathbf{p} = [0\ \ddot{q}]^{\mathrm{T}}$ may be regarded as a disturbance vector. The discrete-time model of the system (7) for the given sampling period T is obtained in the form

$$\begin{aligned} &\mathbf{e}(k+1) = e(k) + T\mathbf{A}_{\delta}\mathbf{e}(k) - T\mathbf{b}_{\delta}u_{sm}(k) + T\mathbf{d}_{\delta}(k), \\ &\mathbf{A}_{\delta} = (\mathbf{A}_d - \mathbf{I})/T,\ \mathbf{b}_{\delta} = \mathbf{b}_d/T,\ \mathbf{d}_{\delta} = \mathbf{d}_d/T \quad (8) \\ &\mathbf{A}_d = e^{\mathbf{A}T};\ \mathbf{b}_d = \int_0^T \mathbf{e}^{\mathbf{A}t}\mathrm{d}t\mathbf{b};\ \mathbf{d}_d = \int_0^T \mathbf{e}^{\mathbf{A}t}\mathbf{p}(kT + T\text{-}t)\mathrm{d}t. \end{aligned}$$

Control task is to annul the tracking error, i.e. the trajectories of the system (8) should reach state space origin. Using the concept of DSMC this would mean that system trajectories from an arbitrary initial point should reach in finite time the sliding line $s(k)$=0, defined by the switching function

$$s(k) = \mathbf{c}_{\delta}\mathbf{e}(k),\ \mathbf{c}_{\delta} = [c_{\delta 1}\ \ c_{\delta 2}], \quad (9)$$

and continue to slide along the line into the origin, which would result in ideal tracking. System dynamics in the sliding mode is strictly defined by the sliding line vector $\mathbf{c}_{\delta}$, which should be chosen according to the desired dynamics.

Sliding line reaching dynamics in [16] is proposed as

$$s(k+1)-s(k)=-\Phi(s),\ \Phi(s)=\min\{|s(k)|,\sigma T\}\operatorname{sgn}(s(k))\ ,\quad(10)$$

which is accomplished according to (9) and (8) by the following control

$$u_{sm}(k)=\mathbf{c}_\delta\mathbf{A}_\delta\mathbf{e}(k)+\mathbf{c}_\delta\mathbf{d}_\delta(k)+T^{-1}\Phi(s) \quad (11)$$

under normalization $\mathbf{c}_\delta\mathbf{b}_\delta=1$. This control is not feasible due to unknown $\mathbf{d}_\delta$, that is $\ddot{q}$. A feasible control

$$u_{sm}(k)=\mathbf{c}_\delta\mathbf{A}_\delta\mathbf{e}(k)+T^{-1}\min\{|s(k)|,\sigma T\}\operatorname{sgn}(s(k)) \quad (12)$$

gives the following reaching dynamics.

$$s(k+1)-s(k)=-\min\{|s(k)|,\sigma T\}\operatorname{sgn}(s(k))+T\mathbf{c}_\delta\mathbf{d}_\delta(k)\ . \quad (13)$$

It is evident that control (12) has two modes: nonlinear and linear. Nonlinear control

$$u_{sm\_n}(k)=\mathbf{c}_\delta\mathbf{A}_\delta\mathbf{e}(k)+\sigma\operatorname{sgn}(s(k)) \quad (14)$$

acts outside the zone $|s(k)|<\sigma T$, which produces the reaching dynamics given by

$$s(k+1)-s(k)=-\sigma T\operatorname{sgn}(s(k))+T\mathbf{c}_\delta\mathbf{d}_\delta(k)\ . \quad (15)$$

To ensure the reaching of the sliding line, the condition $[s(k+1)-s(k)]s(k)<0$ must be satisfied. Under assumption that the reference is a smooth function, its second derivative is bounded $|\ddot{q}|\le M_r$ and therefore the disturbance is also bounded $\|\mathbf{d}_\delta\|\le M$. Reaching is secured if the switching gain σ fulfills inequality $\sigma>\|\mathbf{c}_\delta\|M$. It means that the system trajectories will enter zone $|s(k)|<\sigma T$ in finite number of steps.

Inside this zone, the control signal is linear

$$u_{sm\_l}(k)=\mathbf{c}_\delta\mathbf{A}_\delta\mathbf{e}(k)+T^{-1}s(k)\ , \quad (16)$$

which provides $s(k+1)=T\mathbf{c}_\delta\mathbf{d}_\delta(k)$, indicating that a quasi-sliding mode arises in a single step within a domain described by

$$S_{qs}=\{\mathbf{e}\,\big|\,|s(\mathbf{e})|\le T\,\|\mathbf{c}_\delta\|\,M\}\ . \quad (17)$$

For small sampling periods T the width of the quasi-sliding domain is also small, which guarantees high-precision tracking. If the referent signal is q is a constant or a ramp function, the second component of the disturbance is zero, $\ddot{q}=0$, which gives $\mathbf{p}=\mathbf{b}_d=\mathbf{b}_\delta=M=0$. It yields $s(k+1)=0$, that is, ideal discrete sliding mode occurs in one step that provides ideal tracking of ramp references. The control (16) is the so called equivalent control u_{eq}.

Further tracking improvement was suggested in [14] by introduction of the supplemental integral action with respect to switching variable $s(k)$. Namely, integration is activated only inside the linear control zone, only when the tracking error is small, i.e. in the sliding mode final stage. Activation of the integral action within the nonlinear control zone or distant from the origin is completely unnecessary and may can produce an unwanted overshoot. This idea is described by the following expression

$$u_I(k)=\begin{cases}0, & \|\mathbf{e}(k)\|>\rho>0,\\ hs(k)+u_I(k-1), & \|\mathbf{e}(k)\|\le\rho,\end{cases} \quad (18)$$

where ρ is a small positive constant. Integral gain h should satisfied condition $0<h<1/T$ to preserve system stability.

The resulting control signal as an output of the designed DSM controller, created by merging the described control components, is summarized by

$$u_{sm}(k)=\begin{cases}u_{sm\_n}(k), & |s(k)|>\sigma T,\\ u_{sm\_l}(k), & |s(k)|\le\sigma T,\\ u_{sm\_l}(k)+u_I(k), & |s(k)|<\sigma T\cap\|\mathbf{e}(k)\|<\rho.\end{cases} \quad (19)$$

Because of the introduced additional integral action, which increases tracking accuracy, the designed DSM controller provides ideal tracking of parabolic signals.

Since the main controller is already tuned by some conventional method, it remains to define AC sliding mode dynamics, which is prescribed by the selection of vector $\mathbf{c}_\delta$. In case of a second order system in sliding mode, due to the order reduction of the differential equation that describes sliding mode dynamics, a single eigenvalue $z_1=e^{-\alpha T}$ determines desired system dynamics. The desired slope α of the sliding line is established if it holds

$$\mathbf{c}_\delta\mathbf{b}_\delta=1\quad\wedge\quad c_{\delta1}/c_{\delta2}=\alpha\ . \quad (20)$$

The procedure for the calculation of vector $\mathbf{c}_\delta$ in case of higher order systems is given in [16].

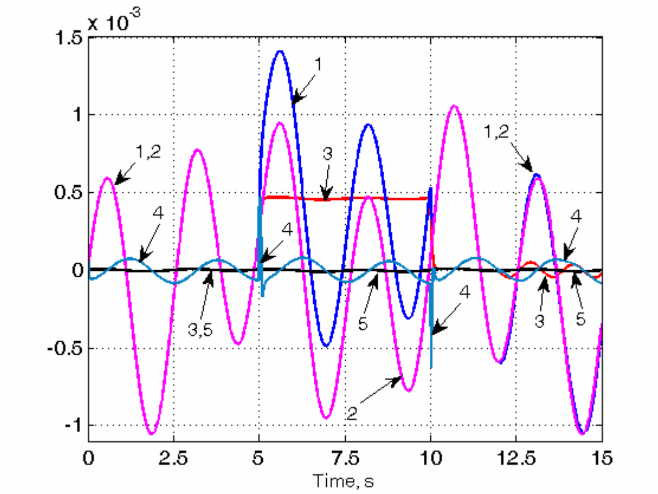
IV. Simulation Example

Permanent magnet direct current motor is considered as a plant, whose nominal mode is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -26.5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 654\end{bmatrix}u+\begin{bmatrix}0\\ 1\end{bmatrix}f,\quad y=x_1\ . \quad (21)$$

Sampling period is $T=0.4$ ms. Parameters of the DSM controllers within ACs are set as: $\alpha=50$; $h=100$ in DC and $h=1000$ in FC; $\sigma=10$; $\rho=0.01$; $\mathbf{c}_\delta=-[7.557248\ 0.151449]\cdot10^{-2}$. Differentiator [12] is employed to obtain the derivatives of the input signals r and q. The main controller is a PD controller that is tuned by the following selection of the well-known parameters: $K_r=25$ and $T_d=1/26.5$ s. The input signal that represents angular position reference is described by $r(t)=5[\cos(t)-\cos(2.5t)]$. Load torque that acts as an external disturbance is expressed by $f(t)=200[h(t-5)-h(t-10)]+$ $+20\sin(5t)h(t-12)$, where $h(t)$ represents step function. Tracking errors obtained by simulations in case of different configurations are given in Fig. 4.

Fig. 4 shows that the performance of the main controller only (line 1) is unsatisfactory both with respect to reference tracking and disturbance rejection. Activation of active DC (line 2) eliminates disturbances whereas tracking remains unchanged. Only the inclusion of active FC (line 5), which ensures almost ideal tracking, results in a superior response comparing with the conventional structure.



Line	Main controller	DC	FC
1	On	Off	Off
2	On	On.	Off
3	On	Off	On
4	Off	On	On
5	On	On	On

Fig. 4. Tracking errors of the proposed system for various configurations.

V. Conclusion

The paper proposes a way to upgrade conventional servo-systems by introduction of digital ACs. Adjoining of DC and FC improves system performances in reference tracking as well as disturbance rejection. Both compensators contain active DSM controlled subsystems, whose controllers are designed for the nominal plant. Simulation results evidently show that the proposed control extension ensures superior performance comparing to the initial system, confirming that a conventional positioning system becomes a robust high-performance tracking system.

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