Multivariable Modeling and Decentralized Robust Linear Controllers for Current-Sharing DC/DC Converters

Aleksandar Ž. Rakić, Trajko B. Petrović

Abstract—In this paper, the multivariable approach is utilized to obtain linear models of current-sharing switching converters. These multivariable linear models are suited for frequency framework of various linear control design techniques. Frequency domain setup is proposed for the robust linear design and all of its elements are defined in order to obtain simple decentralized robust PI/PID type controllers by reduction. The feasibility of the approach is verified for robustness and performance in a three parallel boost converters case study.

Index Terms—modeling, robust control, DC/DC converters, current sharing.

I. INTRODUCTION

HIGH power demands of the consumer electronics, modularity and redundancy requirements often bring out the need of several switching power supplies [1, 2] working in parallel and sharing the current to be supplied to the load.

Linear controllers are dominant and widely accepted for their simplicity, common understanding and clear insight to control design impact on fulfillment of stability and performance requirements.

On the other hand, in current-sharing applications there exists abundance of feedback loops which make problem multivariable i.e. hard for modeling, analysis and control design. Also, linear controllers usually suffer of weak performance or even instability when large scale disturbance or significant set point changes are applied. The idea to overcome the drawbacks of the linear approach to the control design is to derive the appropriate multivariable model of the current-sharing configuration, suitable for the multivariable robust control design.

Multivariable robust linear control design [3-5] will make the system insensitive to the uncertainties of modeling and its industry application is getting wider acceptance nowadays for powerful support in research and development phase of the design, like [6] and [7].

Further fuzzification, resulting in robust fuzzy control, is perspective to improve large signal responses [8].

The purpose of this paper is to utilize the multivariable approach in order to obtain linear models of current-sharing switching converters as the alternative to simplified conventional linear approach to modeling [9-14]. Obtained multivariable linear models are to be suitable for frequency framework of various linear control design techniques and they will be used to obtain robust, but simple PI/PID controllers, applicable according to industrial needs.

The paper is organized in sections. Sect. 2 is the place where the existing multivariable model [15] is presented. The subject of Sect. 3 is the derivation of the proposed full multivariable state-space model. In Sect. 4 the proper framework for the robust linear control design is introduced and discussed. Sect. 5 consists of the control design verification in the case study. The conclusion is presented in Sect. 6.

II. PARALLELING SINGLE UNIT LINEAR MODELS

The power stage of each converter (like the boost given for example in Fig. 1) is a variable structure process, depending on the state of the control switch within the switching period:

- while \( nT \leq t \leq nT + t_{on} \), switch in ON, structure is S1,
- while \( nT + t_{on} \leq t \leq (n+1)T \), switch is OFF, structure is S2.

Introducing: duty ratio \( d_{on} = t_{on} / T \), state variables \( x = [v_{in}, i_i]^T \) and external inputs \( d = [v_{in}, i_i]^T \), the state space models become:

\[
x(t) = A_1 x(t) + E_1 d(t), \quad nT < t < (n + d_{on})T, \tag{1}
\]

\[
x(t) = A_2 x(t) + E_2 d(t), \quad (n + d_{on})T \leq t < (n + 1)T. \tag{2}
\]

State-space averaging gives nonlinear model:

\[
x(t) = Ax(t) + Ed(t),
A = d_{on} A_1 + (1-d_{on}) A_2, \quad E = d_{on} E_1 + (1-d_{on}) E_2 \tag{3}
\]

The control signal, the input signals, the states, and the output signals can be divided into the stationary values and the increments:

\[
d_q = D_q + d, \quad d = D + \hat{d}, \quad x = X + \hat{x}, \quad y = [v_{out}, i_i]^T = Y + \hat{y}. \tag{4}
\]
Unit Linear (SUL) model: linearization of the averaged nonlinear model gives the Single

\[ \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{d} + E\mathbf{\tilde{d}}, \quad \mathbf{y} = C\mathbf{x} + D\mathbf{d} + F\mathbf{\tilde{d}}, \]

where the state-space matrices are given by:

\[ A = A_1D_1 + A_2(1-D_1), \quad B = (A_1-A_2)X+(E_1-E_2)D, \]
\[ C = C_1D_1 + C_2(1-D_1), \quad D = (C_1-C_2)X+(F_1-F_2)D, \]
\[ E = E_1D_1 + E_2(1-D_1), \quad F = F_1D_1 + F_2(1-D_1). \]

Relevant transfer function matrices in the small signal input-output representation of the process:

\[ \mathbf{\hat{y}} = \begin{bmatrix} \mathbf{v}_{\text{out}}(s) \\ i_j(s) \end{bmatrix} = \begin{bmatrix} \mathbf{P}_1(s) \\ \mathbf{P}_2(s) \end{bmatrix} \begin{bmatrix} d_j(s) \\ A_j(s) \end{bmatrix} + \begin{bmatrix} \mathbf{Z}_{\text{out}}(s) \\ \mathbf{T}(s) \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\text{in}}(s) \\ i_j(s) \end{bmatrix}, \]

are defined by the state-space matrices in the following way:

- \( \mathbf{P}_1(s) \) has state-space representation \((A, B, C, D)\),
- \( \mathbf{P}_2(s) \) has state-space representation \((A, E, C, F)\),

where matrices \( A, B, C, D, E \) and \( F \) are given in (9).

Small signal model of the overall process of current-sharing power stages is given by the equations:

\[ v_{\text{out}} = \frac{1}{Y_{\text{load}}} + \sum_{j=1}^{n} \frac{1}{Z_{\text{out},j}} \left( \int P_j d_j + A_j v_{\text{in}} + \frac{1}{Z_{\text{out},j}} \right) i_j, \quad j = 1, \ldots, n \]

where: \( R_{\text{load}} \) is the nominal load, \( P_j, A_j, Y_{\text{in},j}, T_j \) and \( Z_{\text{out},j} \) are: transfer function from control to the output voltage, transfer function from control to the unit’s current, audio susceptibility, input admittance, transconductance and output impedance, all for the \( j \)-th unit, respectively.

Introducing the vectors of the input, the disturbance and the output:

\[ \mathbf{u} = [d_{q1}, d_{q2}, \ldots, d_{q_n}]^T, \quad \mathbf{d} = [v_{\text{in}}, i_{j1}, \ldots, i_{jn}]^T, \]

Paralleled Single Unit Linear (PSUL) model can be presented in the matrix form:

\[ \mathbf{y} = \mathbf{P}_1 \mathbf{u} + \mathbf{P}_2 \mathbf{d}, \]

which involve transfer function matrices:

\[ P_1 = \begin{bmatrix} P_{i1} \ldots P_{im} \end{bmatrix}, \quad P_2 = \begin{bmatrix} \sum_{j=1}^{n} A_j Z^*_{\text{out}} \\ Y_{\text{in},1} T^*_{c1} \\ \vdots \\ Y_{\text{in},n} T^*_{cn} \end{bmatrix}, \]

with the constituting transfer functions:

\[ Z^*_{\text{out}} = \frac{1}{R_{\text{load}}} + \sum_{j=1}^{n} \frac{1}{Z_{\text{out},j}}, \quad P_{i,j} = \frac{Z^*_{\text{out}}}{Z_{\text{out},j}}. \]

III. FULL MULTIVARIABLE STATE SPACE MODEL

Introducing the full state vector of the current-sharing configuration:

\[ \mathbf{x} = [v_{c1}, \ldots, v_{cn}, i_{j1}, \ldots, i_{jn}]^T, \]

and using the disturbance vector \( \mathbf{d} \) defined in (13), Kirchoff’s laws for the overall system can be arranged in the form:

\[ \mathbf{x} = A\mathbf{x} + E\mathbf{d} + A_j \mathbf{x} + E_j \mathbf{d} \] (direct control term)
\[ + A_j \mathbf{x} + E_j \mathbf{d} + A_j \mathbf{x} + E_j \mathbf{d} \] (indirect control term),

where the indexing terms consist of: “\( j \)” – direct influence of \( j \)-th switch control to the \( j \)-th capacitance voltage and \( j \)-th inductor current, “\( k \)” – indirect influence of \( k \)-th switch control to the \( j \)-th capacitance voltage and \( j \)-th inductor current, “\( 1 \)” – influence when switch in ON, “\( 2 \)” – influence when switch in OFF and “\( 0 \)” – influence not altered by the position of switches.

In the similar manner, output equations can be arranged in the form:

\[ \mathbf{y} = C_1 \mathbf{x} + F_1 \mathbf{d} + C_j \mathbf{x} + F_j \mathbf{d} + C_j \mathbf{x} + F_j \mathbf{d}, \]

where the output vector \( \mathbf{y} \) in given in (13).

Multi-input state-space averaging and linearization give Full Multivariable State Space Linear (FMSSL) model:
\begin{equation}
\dot{\mathbf{x}} = A \mathbf{x} + B d \mathbf{q} + E \mathbf{d}, \quad \dot{\mathbf{y}} = C \mathbf{x} + D d \mathbf{q} + F \mathbf{d}, \tag{20}
\end{equation}
where the state space matrices are:
\begin{align*}
A &= A_0 + (A_1 + A_2) D_0 + (A_3 + A_4) (1 - D_0), \\
E &= E_0 + (E_1 + E_2) D_0 + (E_3 + E_4) (1 - D_0), \\
B_1 &= (A_1 - A_2) \mathbf{X} + (E_1 - E_2) \mathbf{D}, \\
B_2 &= (A_3 - A_4) \mathbf{X} + (E_3 - E_4) \mathbf{D}, \\
B &= \text{diag}(B_1) + \text{offdiag}(B_2), \\
C &= C_0 + C_1 D_0 + C_2 (1 - D_0), \\
F &= F_0 + F_1 D_0 + F_2 (1 - D_0), \\
D_j &= (C_1 - C_2) \mathbf{X} + (F_1 - F_2) \mathbf{D}, \\
D &= \text{diag}(D_j).
\end{align*}

(21)

For the purposes of the comparison with PSUL and the notation in the control framework, transfer function matrix representation of FMSSL model is given also in the form of (14), where:
- \( \mathbf{P}_1(s) \) has state-space representation \((A, B, C, D)\),
- \( \mathbf{P}_2(s) \) has state-space representation \((A, E, C, F)\),
and matrices \(A, B, C, D, E\) and \(F\) are given in (21).

**IV. FREQUENCY DOMAIN FRAMEWORK FOR THE ROBUST LINEAR DESIGN**

Since the output vector \(\mathbf{y}\) is of dimension \(n+1\) and there are only \(n\) independent input switch control signals, the transfer function matrix \(\mathbf{P}_1\) is not square. One way to make it square, in order to obtain a closed-loop control, is to redefine the outputs to represent the output voltage and the current distribution between the units [12]:
\begin{equation}
\mathbf{y}' = \begin{bmatrix} \mathbf{v}_{\text{out}} & \Delta i_{i2} & \Delta i_{i3} & \cdots & \Delta i_{in} \end{bmatrix}^T, \quad \Delta i_i = i_i - i_{i1}. \tag{22}
\end{equation}

The transformation matrix:
\begin{equation}
\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\
0 & -1 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & -1 & 0 & 0 & \cdots & 1 \end{bmatrix}_{\text{act}(n+1)} \tag{23}
\end{equation}
introduces the current difference between the \(i\)-th unit and the reference (master) unit 1, making the squared plant \(\mathbf{S}\mathbf{P}_1\) suitable for control. Moreover, master-slave (M-S) control configuration can be represented by matrix equation:
\begin{equation}
\mathbf{u} = \mathbf{S} \mathbf{K} \mathbf{e}, \quad \mathbf{K} = \text{diag}(\mathbf{K}_1, \mathbf{K}_2, \ldots, \mathbf{K}_n), \tag{24}
\end{equation}
where: error vector \(\mathbf{e}\) is the input to controller \(\mathbf{K}\), and the introduced matrix
\begin{equation}
\mathbf{S}_e = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 \\
1 & 0 & 1 & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
1 & 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}_{\text{act}(n+1)} \tag{25}
\end{equation}
makes generalized plant:
\begin{equation}
\mathbf{P}_e = \mathbf{S}_e \mathbf{P}_e \tag{26}
\end{equation}
suitable for decoupled consideration of voltage and current control. Namely, first channel of \(\mathbf{P}_e\) corresponds to the nominal plant for the voltage control:
\begin{equation}
G_v(s) = \mathbf{P}_e(1,1), \tag{27}
\end{equation}
and all the other diagonal channels represent nominal plants for the control of the current loops:
\begin{equation}
G_i(s) = \mathbf{P}_e(i,i), \quad i \neq 1. \tag{28}
\end{equation}

The block diagram of the robust linear (RL) control design setup is presented in Fig. 2, with following signals denoted: \(r\) – the reference (set-point) signal, \(d\) – the disturbance signal, \(e = r - y\) is the error in reference tracking, \(e'\) – the performance weighted error, and \(u\) – the control signal. Relevant transfer functions are: \(G(s)\) – the nominal linear model of the plant, \(G_d(s)\) – the disturbance model, \(K(s)\) – the linear robust controller to be designed, \(W_i(s)\) – the multiplicative input uncertainty bound (uncertainty weighting function), \(\Delta(s)\) – an unknown but unity-normed multiplicative uncertainty of modeling, and \(W_p(s)\) – the performance weighting function.

![Fig. 2. Block Diagram of Robust Linear (RL) Control Setup.](image-url)
\[ \gamma_{\mu/\omega} = \min_{k(s)} \left\| W_p G(s) \right\|_{\mu/\omega} = \min_{k(s)} \left\| W_p G(s) (1+G)^{-1} \right\|_{\mu/\omega} \]  
\[ \text{is carried out, while for the disturbance rejection adequate minimization is:} \]
\[ \gamma_{\nu/\omega} = \min_{k(s)} \left\| W_p S(s) \right\|_{\nu/\omega} \]
\[ = \min_{k(s)} \left\| W_p G(s) (1+G)^{-1} \right\|_{\nu/\omega}. \]

The choice of performance weighting function \( W_p \) forces the \( \mu/\omega_k \) controller to be in the form of causal PI proportional-integral (PI) or causal proportional-integral-derivative (PID) controller, plus the higher order dynamics. When the control design is obtained, zero-pole cancellation should be applied and dynamics much higher than bandwidth should be neglected, so the final reduced robust linear controllers will be in the form of causal PI:
\[ K_{RL}^{\mu}(s) = \frac{K_0 (s/\omega + 1)}{s(s/\omega + 1)} \]  
or causal PID:
\[ K_{RL}^{\nu}(s) = \frac{K_0 (s^2 / \omega^2 + 2\zeta s/\omega + 1)}{s(s/\omega + 1)} \]
where \( K_0 \) is the velocity constant of the controller and the parameters \( \omega_c \) and \( \omega_p \) are natural frequencies of the controller’s zero and pole.

V. CASE STUDY VERIFICATION OF THE PROPOSED DESIGN

In order to verify the proposed design, three boost converters working in parallel will be considered with the following parameters: \( f_{sw} = 200kHz \), \( V_{IN} = 18V \), \( V_{OUT} = 25V \), \( R_{load} = 4.17\Omega \), \( L = 41.67\mu H \), \( R_L = 50m\Omega \), \( C = 26.67\mu F \), \( R_c = 25m\Omega \). Due to space limitation, the state space matrices of PSUL and FMSSL models of the case study plant are omitted.

In Fig. 3, PSUL and FMSSL models are compared for small signal transfer functions from the \( j \)-th control to the outputs: the regulated voltage \( V_{out} \) and the \( j \)-th inductor current. Significant differences are found in the high frequency region and they are presumed to be the result of the PSUL’s neglected multivariable dynamics. This conclusion is supported by the comparison in Fig. 4 of the FMSSL’s transfer functions with the same ones obtained in Spice with the Voltage Mode Large Signal Continuous Conduction Mode (VMLSCCM) model of the switches. The gain of the FMSSL is the good model of the multivariable interconnection i.e. the transfer function from the \( k \)-th control to the \( j \)-th inductor current (Fig 4, bottom), fully neglected in PSUL modeling. In Fig. 5, FMSSL’s disturbance transfer functions are also found to fit the behavior of the realistic Spice model. Therefore, the following controller design will be based exclusively on the FMSSL model.

The bandwidths of the robust linear controllers are adopted to be: 5 kHz for the voltage loop and 20 kHz for the current loop. Obtained \( \mu/H_\nu \) controllers are reduced to:
\[ K_{RL}^{\mu}(s) = \frac{515.8(s^2/(1.373\cdot10^5)^2 + s/(1.14\cdot10^4) + 1)}{s(s/(2.854\cdot10^5) + 1)} \]  
\[ K_{RL}^{\nu}(s) = \frac{227.5(s/1330 + 1)}{s(s/(4.59\cdot10^4) + 1)} \]

The verification of the control design large is conducted through simulation [6].
In order to test robustness of the control to the tolerance of the components, the parameters of the units in a three boost setup are perturbed in the following way:

- the gain of the voltage loop is increased by 20%,
- control-to-current gain of unit #2 is increased by 20%,
- control-to-current gain the unit #3 is decreased by 20%.

The first experiment was the load change step from full load to 50% (from 3x2A to 3x1A) and the unit currents along with voltage waveform and the control signals (duty ratios) are presented in Fig. 6. No significant deterioration of the perturbed plant case comparing to nominal is observed.

![Fig. 4. Small signal transfer functions: top – from control to the output voltage, middle – from control $q_j$ to inductor current $i_{Lj}$, bottom – from control $q_k$ to inductor current $i_{Lj}$, (Spice VMLSCCM – black, FMSSL – gray).](image)

![Fig. 5. Small signal transfer function matrix from the disturbance vector to the output voltage and one of the inductor currents (Spice VMLSCCM – black, FMSSL – gray).](image)
The second experiment was the voltage supply step change in amount of $v_{in} = 1\text{V}$, where the open-loop voltage drop effect would be approx. $v_{in}(1-d_Q) \approx 1.39\text{V}$. It can be seen in Fig. 7, as well as in the previous experiment, disturbance is eliminated in the short time and with the desirable waveforms and amplitudes of the control signals.

VI. CONCLUSION

In this paper, multivariable modeling approach is utilized to obtain new linear state-space model of the current-sharing converters. Benefits of the proposed FMSSL are in the field of interactions in the multivariable plant of current-sharing converters, neglected in existing PSUL model. FMSSL shows good agreement with the small signal multivariable behavior obtained by the Spice model.

Proposed FMSSL model is derived in the state-space, so it is the minimal realization. As such, it is suitable for the various control designs resulting in controllers of minimal order.

As a perspective candidate for maximization of robustness and performance, robust control design is carried out and optimal robust linear controllers are obtained and they are reduced to simple decentralized PI and PID type controllers. Proposed robust design is successfully verified in the case study with three paralleled boost converters.

Further research will be directed towards the analysis of the FMSSL robustness properties (uncertainty bounds), application of the proposed model in the appropriate control designs and extension of the proposed multivariable approach to the random switching schemes for electromagnetic emission reduction in distributed current-sharing applications.
References


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