

# Calibration of High Voltage Linear Voltmeter

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**Abstract**—In this paper, linear voltmeter for direct measuring on 20 kV voltage level is described. Its main feature is linearity, what is verified theoretically and experimentally.

**Index Terms**—harmonic measuring instrument, linearity, voltmeter, testing.

## I. INTRODUCTION

NOWADAYS, high voltage measurement is mainly made by high voltage measuring transformers which adapt measured voltage to measurement range of instrument, and after that by the instrument itself – with voltmeter. On very high voltages (over 110 kV), capacitive dividers are used. The key element is the voltage transformer. Recently, the high voltage resistor divider has appeared.

On low voltage side (0.4 kV), especially in electric energy meters, voltage resistor dividers are used for voltage measurement. Both these solutions, voltage resistor dividers and voltage measuring transformers, in addition to useful features being ground of application, they have some disadvantages:

- a) Voltage measuring transformer is nonlinear
- b) With voltage resistor divider there is no galvanic separation of the instrument from the measured voltage.

In the paper [1], it is putting forward as a measuring method of net voltage which is overcoming the both mentioned disadvantages: stochastic measuring method. It uses series resistor at the input and transformer without core (mutual inductance) so the measurement is completely linear and the galvanic separation of the input from the measuring instrument is accomplished.

The coreless transformer has a very feeble output signal, in other words, it has a low signal/noise ratio, however, in literature [2] it is demonstrated that the stochastic method can overcome this problem successfully and the noise is being firmly suppressed independently on probability distribution function. In addition, the applied measuring method eliminates the serious problem appearing with high voltage resistor divider – dependence of dividing factor on input voltage. The resistor is simply in function of series resistor here and there is no voltage dividing.

In the second chapter, the built linear voltmeter is described as it follows:

in A. the principle of functioning of linear voltmeter is described, and in B. the principle of functioning of Harmonic Measuring Instrument (HMI), which is explained in details in [3].

In Chapter 3, the problem of experimental testing of metrological properties of built linear high voltage voltmeter is described, as well as the testing itself.

In Chapter 4, the testing procedure is discussed briefly, and in 5. a short conclusion is given.

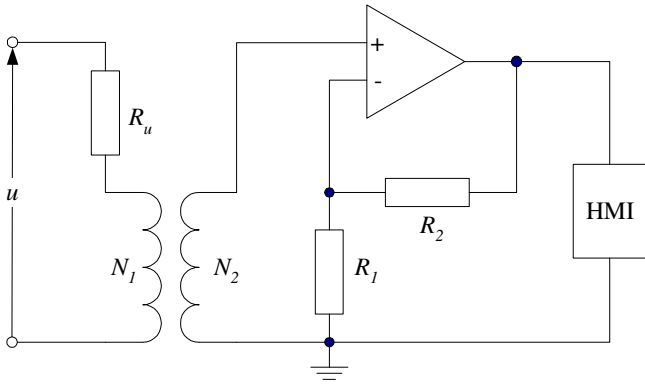
## II. LINEAR VOLTMETER

### A. Proposal of New Linear Voltmeter

The list of denotations:

- $U$  – measured input voltage,
- $R_u$  - series resistor,
- $u_2$  - secondary winding voltage,
- $e_2$  - induced electromotive force on secondary winding,
- $U_2$  - RMS value of secondary voltage,
- $L_1, L_2, L_{12}$  - primary inductivity, secondary inductivity, mutual inductivity,
- $I$  - RMS value of primary current,
- $\omega$  – angular frequency of fundamental harmonic,
- $E_1$  - RMS value of electromotive force on primary winding,
- $E_2$  - RMS value of electromotive force on secondary winding,
- $M$  – dimensionless constant,
- $u_{2i}$  - amplitude of  $i^{\text{th}}$  harmonic on secondary,
- $U_i$  - amplitude of  $i^{\text{th}}$  harmonic of input voltage  $u$ ,
- $\varphi_{2i}$  - phase of  $i^{\text{th}}$  harmonic on secondary,
- $\varphi_i$  - phase of  $i^{\text{th}}$  harmonic of input voltage  $u$ ,
- $\Gamma_{NA}$  - relative error (nonlinearity) of amplitude,
- $\Gamma_{NF}$  - relative error (nonlinearity) of phase.

The proposed linear voltmeter is showed on Fig. 1.



(HMI – Harmonic Measuring Instrument )

Fig. 1. Scheme of new linear voltmeter.

The absence of core in this transformer, showed in this picture, allows the instrument to be linear and there is also galvanic separation from measuring circuit.

$\bar{U}_{2i}$  is  $i^{\text{th}}$  harmonic of input voltage on amplifier in complex form and it is given with expression

$$\bar{U}_{2i} = \frac{ji\omega L_{12}}{R_u + ji\omega L_1} \cdot \bar{U}_i \quad (1)$$

where  $\bar{U}_i$  represents  $i^{\text{th}}$  harmonic of phase-to-phase voltage in complex form.

It is supposed that the taken voltage value is  $U=600$  V and the primary current is (current in primary winding)  $I=1$  mA.

Because of absence of core the inductivities  $L_1$  and  $L_2$  are considerable small, and  $I = \frac{U}{R_u} \Rightarrow R_u = 600\text{k}\Omega$ .

Current in secondary winding because of amplifier coupling is tending to zero, so we have  $I_2=0$ .

Suppose that  $N_1 = N_2 = 2000$  turns and the diameter of each winding is  $r=1$  cm.

Intending to get  $U_2 = E_2$  we suppose that  $L_1 \approx L_2$ ,  $L_{12} \approx \sqrt{L_1 L_2} \approx L_1$ , where  $L_1$ ,  $L_2$  i  $L_{12}$  are inductivities of primary and secondary windings and mutual inductivity.

In case of sine current the electromotive force is:

$$E_2 = \frac{\mu_0}{2} \cdot N_1 N_2 r \pi \omega I \quad (2)$$

If the frequency  $f=50\text{Hz}$ , than the electromotive force is  $E_2: E_2 \approx 25 \cdot 10^{-3}$  V.

As mentioned  $L_1 \approx L_2 \approx L_{12}$ , it succeeds that:

$$\frac{E_2}{U} \approx \frac{E_1}{U} = \frac{\omega L_1}{\sqrt{R_u^2 + \omega^2 L_1^2}}, \text{ and} \quad (3)$$

$$E_1 = \frac{\omega L_1}{\sqrt{R_u^2 + \omega^2 L_1^2}} \cdot U \approx E_2 \quad (4)$$

Generally, in case of  $i^{\text{th}}$  harmonic:

$$\bar{U}_{2i} = \frac{ji\omega L_{12}}{R_u + ji\omega L_1} \cdot \bar{U}_i \approx \frac{ji\omega L_1}{R_u + ji\omega L_1} \cdot \bar{U}_i \quad (5)$$

where  $\bar{U}_i$  and  $\bar{U}_{2i}$   $i^{\text{th}}$  harmonics of input voltage and to it corresponding secondary voltage.

From expression (5) we get:

$$\frac{U}{E_2} \approx \frac{U}{E_1} \approx \frac{R_u}{\omega L_1} \approx M \approx 24000 \frac{\pi}{4} \quad (6)$$

In case of sine voltage and fundamental frequency  $f=50\text{Hz}$ .

The electromotive force  $e_2$  is considerable small and it should be augmented into voltage range of  $\pm 2.5\text{V}$  amplifying k times.

Basing on expression (6):

$$\bar{U}_{2i} = \frac{ji \frac{R_u}{M}}{R_u + ji \frac{R_u}{M}} \cdot \bar{U}_i = \frac{ji}{M + ji} \bar{U}_i \quad (7)$$

Amplitude of  $i^{\text{th}}$  harmonic on secondary is given by expression:

$$U_{2i} = \frac{i}{\sqrt{M^2 + i^2}} \cdot U_i \quad (8)$$

Its phase is:

$$\varphi_{2i} = i \cdot \frac{\pi}{2} - \arctan \frac{i}{M} + \arg(\bar{U}_i) \quad (9)$$

When  $M \gg i_{\text{max}}$ , including  $i_{\text{max}} = 50$ , the next expression is given by:

$$U_{2i} \approx \frac{i}{M} \cdot U_i, i \quad (10)$$

$$\varphi_{2i} \approx i \frac{\pi}{2} - \frac{i}{M} + \varphi_i \quad (11)$$

Expressions (10) and (11) are showing fundamental linear relation between amplitude and phase of  $i^{\text{th}}$  harmonics of voltage on secondary winding  $u$  represented in figure 1.

Expressions (10) and (11) are very suitable for calibration than they are linear, but there is a question of linearity limits?

The response is given by quadratic member in expansion of expressions (8) and (9). In utility (distribution) systems 50 harmonics are significant,  $i_{\text{max}} = 50$ .

As mentioned above, it is supposed

$$\frac{1}{\sqrt{M^2 + i^2}} \approx \frac{1}{M} \quad (12)$$

for the amplitude and the phase angle

$$\tan \frac{i}{M} \approx \frac{i}{M} \quad (13)$$

Expression can be showed as:

$$\frac{1}{M} \left( 1 - \frac{1}{2} \cdot \frac{i^2}{M^2} \right) \approx \frac{1}{M} \quad (14)$$

Relative error because of previous assumption gives nonlinearity:

$$\Gamma_{NA} = \frac{i^2}{2M^2} \leq \frac{i_{\max}^2}{2M^2} = 0.000\ 002\ 2 \quad (15)$$

Which is not bigger than 2,2 ppm.

Expression (13) can be represented as

$$\frac{i}{M} \cdot \left(1 + \frac{1}{3} \cdot \frac{i^2}{M^2}\right) \approx \frac{i}{M} \quad (16)$$

Relative error because of previous assumption gives nonlinearity:

$$\Gamma_{NF} = \frac{i^2}{3M^2} \leq \frac{i_{\max}^2}{3M^2} = 0.000\ 001\ 5 \quad (17)$$

which is not bigger than 1,5 ppm.

### B. Integrated Instrument for Measurement of Harmonics

Integrated instrument for measurement of harmonics (HMI) is quoted in section A.

This paper gives its most important features. Its scheme is showed in the Fig. 2.

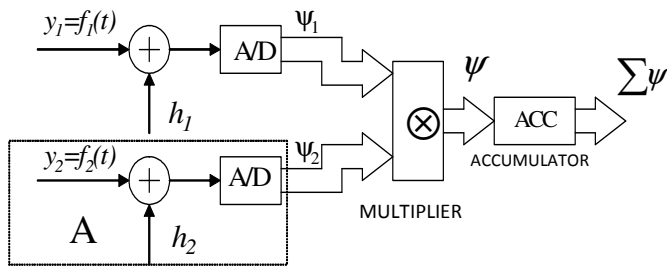


Fig. 2. Integrated instrument for measurement of harmonics.

In the realized instrument, block A is replaced by a memory block which contains samples of basic functions, for example  $y_2 = f_2(t) = R \cos i\omega t$ , and its output value is  $\Psi$ . After one period of fundamental frequency we get

$$\bar{\Psi} = \frac{R}{2} a_i, \text{ where } a_i = \frac{2}{T} \int_0^T f_1(t) \cos i\omega t dt, (i = 0, 1, 2, \dots, m).$$

If the memory unit contains  $y_2 = f_2(t) = R \sin i\omega t$ , than the output is  $\bar{\Psi} = \frac{R}{2} b_i$  i  $b_i = \frac{2}{T} \int_0^T f_1(t) \sin i\omega t dt$ .

In our HMI, resolution of A/D converter is 6 bits. Resolution of basic functions is 8 bits, range R (range of converter) is  $\pm 2,5V$ , the frequency is 250kHz and the frequency of measured signal is  $f=50Hz$ .

It should be noticed that:

$$U_i = \sqrt{a_i^2 + b_i^2}, \text{ and} \quad (18)$$

$$\varphi_i = \arctan \frac{b_i}{a_i} \quad (19)$$

HMI measures  $a_{im}$  and  $b_{im}$ . Voltage  $u_2$  (secondary voltage) is amplified k times, so:

$$U_{2i} = \frac{\sqrt{a_{im}^2 + b_{im}^2}}{k} = \frac{i}{M} \cdot U_i \quad (20)$$

$$U_i = \frac{M}{i \cdot k} \sqrt{a_{im}^2 + b_{im}^2} = \frac{M}{i \cdot k} \cdot U_{2i} \quad (21)$$

And the phase is :

$$\varphi_i = \arctan \frac{b_{im}}{a_{im}} - i \cdot \left(\frac{\pi}{2} - \frac{1}{M}\right) \quad (22)$$

On the other hand RMS value of measured voltage is given by:

$$U = k_p \cdot \sqrt{\frac{U_{1m}^2}{1^2} + \frac{U_{2m}^2}{2^2} + \dots + \frac{U_{nm}^2}{n^2}} \quad (23)$$

HMI instrument measures voltage derivative and shows that  $i^{\text{th}}$  harmonic is augmented  $i$  times, as the expression (21) shows, each harmonic should be divided by  $i$ .

That means, taking the result into consideration, that the upper limit of absolute error of each measured harmonic is constant and independent on the order of harmonic wave form [3] (measurement error of  $i^{\text{th}}$  harmonic is  $i$  times smaller), that the higher harmonics can be measured more precisely.

### III. PROBLEMS OF TESTING

According to the fact that The Faculty of Technical Sciences do not have high voltage laboratory, testing of the high voltage voltmeter popped up as a serious problem. The first problem is solved by getting high voltage from transformer oil testing device "RR Zavodi Niš - Munja".

The second problem was the accurate measurement of high voltage. For this purpose, high voltage electrostatic voltmeter "S 196" is used. It has class 1, measures RMS value and has quadratic scale. The measurement reach is 30 kV, and has three ranges 7,5kV; 15kV; 30kV. The authors opted for range of 15kV, namely 11,5kV in regard to earth.

The series resistor is assembled from carbon layer resistors 470k $\Omega$ , so the current which flows through primary winding for full range of 11,5kV makes about 1mA.

Voltage amplifier with amplification factor equal to 51 is put on the output of secondary, so the voltage on that instrument, for full range, is 1,085V RMS, the value of amplitude on the input of instrument is 1,085·1,14 $\approx$ 1,53V accordingly.

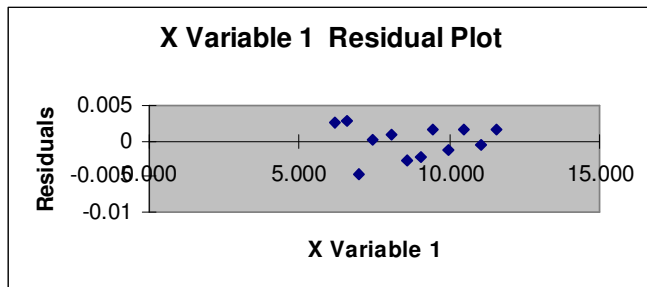
We noticed that resistors closer to "higher" end are warmer, that leads to conclusion that they have greater resistance. It is clear that carbon layer resistors cannot be used as high voltage dividers. When the measurement has held on longer, especially, with higher values (about 10kV), the reading was instable and that is the reason not to use carbon layer resistors as series resistor. The only solution is to put new, stable, high voltage measuring resistors, which are advertised on internet, but our attempts to get them were unsuccessfully.

The purpose of this paper was to confirm the usability of this measuring method on high voltage and during 30 minutes came out the correctness of results listed in this paper.

TABLE I  
MEASURING RESULTS

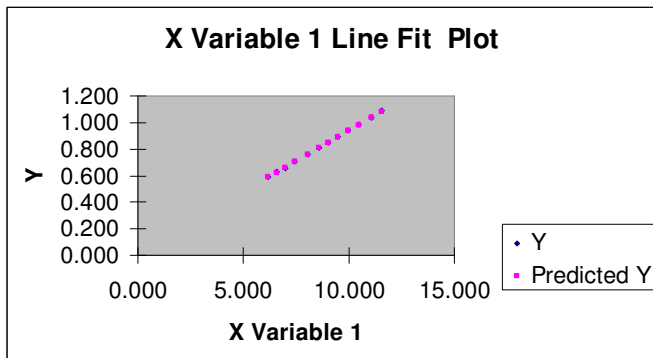
RESIDUAL OUTPUT		
Observation	Predicted Y	Residuals
1	0.587398	0.002602
2	0.625066	0.002934
3	0.662733	-0.00473
4	0.705913	8.66E-05
5	0.761037	0.000963
6	0.810648	-0.00265
7	0.850153	-0.00215
8	0.891496	0.001504
9	0.939269	-0.00127
10	0.982449	0.001551
11	1.038491	-0.00049
12	1.085346	0.002602

Observation – input voltage  $u$  in kV; Predicted Y – output voltage in V;  
Residuals – measurement error in V.



(Residuals – measurement error in V;  $x$  – input voltage  $u$  in kV)

Fig. 3. Graphical presentation of calibration results.



(Y – output voltage in volts;  $x$  – input voltage  $u$  in kV)

Fig. 4. Measurement results – scaled curve.

#### IV. DISCUSSION

The standard instrument has class 1, so the permissible reading error is 0.1%, and the standard deviation is (as a result of supplying the device from the mains –without stabilizer) about 0.5% during the measurement, hence arise that the measurement uncertainty is about 0.6%, that is in great extent in keeping with won nonlinearity. It is possible, that the nonlinearity is still higher. We are pointing out that the scale of instrument „S 196“ is quadratic, so the reading error is 0.1% on the end of the scale, but on the half the scale is 0.4%, so that fact also confirms the conclusion that the linearity of the built instrument is better than the gained 0.5%.

#### V. CONCLUSION

In the paper, linear voltmeter for direct measurement of voltage on 20 kV net (11,5kV regarding to earth).

As the theory confirms and the experiment shows, it has high linearity.

The theory provides linearity from several ppm, while the experiment has showed linearity of 0.5 % , what is on the level of estimated measurement uncertainty for the applied measuring equipment. The result is encouraging and the investigation is going to be continued.

#### REFERENCES

- [1] B. Santrač, M. Sokola, Z. Mitrović, I. Župunski, V. Vujičić "A Novel Method for Stochastic Measurement of Harmonics at Low Signal – to – Noise Ratio", IEEE Transactions on Instrumentation and Measurement, 2008, (accepted for publication)
- [2] D. Čomić, S. Milovančev, V. Vujičić, "A New Approach to Voltage Measurements in Power System", 9<sup>th</sup> International Conference on Electrical Power Quality and Utilisation, Barcelona, October 2007
- [3] N. Pjevalica "Merenje na elektrodistributivnoj mreži u frekvencijskom domenu", Doktorska teza, Fakultet tehničkih nauka u Novom Sadu, 2007.